# STA 313F04 Formulas

Here are some useful formulas. You may use any them without proof, unless you are explicitly asked to derive it. You will not need them all.

- If $\mathbf{A}$ is $n \times r$ and $\mathbf{B}$ is $r \times m$, then $\mathbf{AB} = [\sum_{=1}^{r} a_{ik} b_{kj}]$.
- $E(\mathbf{AXB}) = \mathbf{A}E(\mathbf{X})\mathbf{B}$
- $V(\mathbf{X}) = E\left((\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{X} - \boldsymbol{\mu}_x)'\right)$
- $C(\mathbf{X}, \mathbf{Y}) = E\left((\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{Y} - \boldsymbol{\mu}_y)'\right)$
- The multivariate normal density is $f(\mathbf{x}) = \frac{1}{|\boldsymbol{\Sigma}|^{\frac{1}{2}}(2\pi)^{\frac{p}{2}}} \exp\left[-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu})\right]$.
- If $\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and $\mathbf{A}$ is a matrix of constants, $\mathbf{AX} \sim N(\mathbf{A}\boldsymbol{\mu}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}')$.
- For the multivariate normal, $\widehat{\boldsymbol{\mu}} = \overline{\mathbf{x}}$ and $\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}_i - \overline{\mathbf{x}})(\mathbf{x}_i - \overline{\mathbf{x}})'$
- For the multivariate normal, $-2\log L(\widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\Sigma}}) = n \log|\widehat{\boldsymbol{\Sigma}}| + np[1 + \log(2\pi)]$.

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The EQS model is

$$\boldsymbol{\eta} = \boldsymbol{\beta}\boldsymbol{\eta} + \boldsymbol{\gamma}\boldsymbol{\xi},$$

where $\boldsymbol{\eta}$ is a vector of endogenous variables, $\boldsymbol{\xi}$ is a $N(\mathbf{0}, \boldsymbol{\Phi})$ vector of exogenous variables, and $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are matrices of constants, with $(\mathbf{I} - \boldsymbol{\beta})$ non-singular.

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The LISREL model is

$$\begin{aligned} \boldsymbol{\eta} &= \boldsymbol{\beta}\boldsymbol{\eta} + \boldsymbol{\Gamma}\boldsymbol{\xi} + \boldsymbol{\zeta} \\ \mathbf{y} &= \boldsymbol{\Delta}_y\boldsymbol{\eta} + \boldsymbol{\epsilon} \\ \mathbf{x} &= \boldsymbol{\Delta}_x\boldsymbol{\xi} + \boldsymbol{\delta} \end{aligned}$$

where

$\boldsymbol{\eta}$ is a vector of latent endogenous variables.

$\boldsymbol{\xi}$ is a vector of latent exogenous variables.

$\mathbf{x}$ is a vector of manifest indicators for $\boldsymbol{\xi}$.

$\mathbf{y}$: is a vector of manifest indicators for $\boldsymbol{\eta}$.

$\boldsymbol{\zeta}$, $\boldsymbol{\epsilon}$ and $\boldsymbol{\delta}$ are vectors of error terms, independent of each other and of $\boldsymbol{\xi}$.

All random vectors have expected value zero.

$V(\boldsymbol{\xi}) = \boldsymbol{\Phi}$, $V(\boldsymbol{\zeta}) = \boldsymbol{\Psi}$, $V(\boldsymbol{\epsilon}) = \boldsymbol{\Theta}_\epsilon$, $V(\boldsymbol{\delta}) = \boldsymbol{\Theta}_\delta$.

$\boldsymbol{\beta}$, $\boldsymbol{\Gamma}$, $\boldsymbol{\Delta}_y$ and $\boldsymbol{\Delta}_x$ are matrices of constants, with the diagonal of $\boldsymbol{\beta}$ zero.