

## STA 312F2007 Solutions to Quiz 9

1. (a)

$$\begin{aligned} Cov(X, Y_5) &= E(XY_5) \\ &= E[(\xi + \delta)(\gamma_5\xi + \zeta_5)] \\ &= E(\gamma_5\xi^2 + \xi\zeta_5 + \gamma_5\delta\xi + \delta\zeta_5) \\ &= \gamma_5E(\xi^2) + E(\xi\zeta_5) + \gamma_5E(\delta\xi) + E(\delta\zeta_5) \\ &= \gamma_5Var(\xi) + E(\xi)E(\zeta_5) + \gamma_5E(\delta)E(\xi) + E(\delta)E(\zeta_5) \\ &= \gamma_5\phi \end{aligned}$$

(b)

$$\begin{aligned} \gamma_5 &= \frac{\sigma_{16}}{\phi} \\ \psi_5 &= \sigma_{66} - \gamma_5^2\phi \end{aligned}$$

The objective is to prove that a unique solution exists by showing how to get it. An exact statement of the solution is not necessary.

2.  $H_0 : \gamma_1 = 0$   
 $H_a : \gamma_1 \neq 0$

Test statistic,  $z^* = -7.4615$

Decision rule: reject  $H_0$  if  $|z^*| > 1.96$

Since  $|z^*| > 1.96$ , we reject  $H_0$ .

Fitted equation:  $\text{birthrat} = -0.2329 * \text{Fgnp}$

We conclude that in richer countries, the birth rate is lower.