STA 312F2007 Solutions to Quiz 7

Poisson(λ). $p(y) = \frac{e^{-\lambda}\lambda^y}{y!}$, $y = 0, 1, 2, \ldots$, $\lambda > 0$.

1. $\Theta = \{(\lambda_1, \lambda_2) : \lambda_1 > 0, \lambda_2 > 0\}$

2. $H_0 : \lambda_1 = \lambda_2 \Rightarrow \Theta_0 = \{(\lambda_1, \lambda_2) : \lambda_1 = \lambda_2 > 0\}$

3. restricted: $\hat{\hat{\lambda}}_1 = \hat{\hat{\lambda}}_2 = \hat{\hat{\lambda}} = \frac{\Sigma x + \Sigma y}{n_1 + n_2} = \frac{n_1\bar{x} + n_2\bar{y}}{n_1 + n_2}$

   unrestricted: $\hat{\lambda}_1 = \bar{x}$, $\hat{\lambda}_2 = \bar{y}$

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\begin{aligned}
\mathcal{L}(\lambda) &= \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{y_i}}{y_i!} = \frac{e^{-n\lambda}\lambda^{\sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n}(y_i!)} \\
l(\lambda) &= \log \mathcal{L}(\lambda) = -n\lambda + \sum_{i=1}^{n} y_i \log\lambda - \log\prod_{i=1}^{n}(y_i!) \\
-2l(\lambda) &= 2n\lambda - 2\sum_{i=1}^{n} y_i \log\lambda + 2\log\prod_{i=1}^{n}(y_i!) = 2\left[n\lambda - \sum_{i=1}^{n} y_i\log\lambda + \log\prod_{i=1}^{n}(y_i!)\right]
\end{aligned}
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\begin{aligned}
G &= -2\log\frac{\mathcal{L}(\hat{\hat{\lambda}}_1, \hat{\hat{\lambda}}_2)}{\mathcal{L}(\hat{\lambda}_1, \hat{\lambda}_2)} \\
&= \left[-2\log\mathcal{L}(\hat{\hat{\lambda}}_1, \hat{\hat{\lambda}}_2)\right] - \left[-2\log\mathcal{L}(\hat{\lambda}_1, \hat{\lambda}_2)\right] \\
&= 2\left[(n_1+n_2)\frac{\Sigma x + \Sigma y}{n_1+n_2} - (\Sigma x + \Sigma y)\log\frac{\Sigma x + \Sigma y}{n_1+n_2} + \log\prod_{i=1}^{n_1}(x_i!) + \log\prod_{i=1}^{n_2}(y_i!)\right] \\
&\quad -2\left[n_1\bar{x} - \Sigma x\log\bar{x} + \log\prod_{i=1}^{n_1}(x_i!)\right] - 2\left[n_2\bar{y} - \Sigma y\log\bar{y} + \log\prod_{i=1}^{n_2}(y_i!)\right] \\
&= 2\left[n_1\bar{x}\log\bar{x} + n_2\bar{y}\log\bar{y} - (n_1\bar{x} + n_2\bar{y})\log\frac{n_1\bar{x} + n_2\bar{y}}{n_1+n_2}\right]
\end{aligned}
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4. $n_1 = 60$, $n_2 = 40$, $\bar{x} = 4.733$ and $\bar{y} = 9.35$

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\begin{aligned}
G &= 2\,[(60)(4.733)\log(4.733) + (40)(9.35)\log(9.35) \\
&\quad -[(60)(4.733) + (40)(9.35)]\log\frac{(60)(4.733) + (40)(9.35)}{60+40}\bigg] \\
&= 75.7
\end{aligned}
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