**STA 312F2007 Solutions to Quiz 6**

$$\sum_{i=1}^{n}\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)'\boldsymbol{\Sigma}^{-1}\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)$$

$$=tr\left[\sum_{i=1}^{n}\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)'\boldsymbol{\Sigma}^{-1}\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)\right] \qquad \text{since dimension is } 1\times 1$$

$$=\sum_{i=1}^{n}tr\left[\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)'\boldsymbol{\Sigma}^{-1}\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)\right] \qquad \text{since } tr(\mathbf{A}+\mathbf{B})=tr(\mathbf{A})+tr(\mathbf{B})$$

$$=\sum_{i=1}^{n}tr\left[\boldsymbol{\Sigma}^{-1}\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)'\right] \qquad \text{since } tr(\mathbf{AB})=tr(\mathbf{BA})$$

$$=tr\left\{\boldsymbol{\Sigma}^{-1}\sum_{i=1}^{n}\left[\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)'\right]\right\} \qquad \text{since } tr(\mathbf{A})+tr(\mathbf{B})=tr(\mathbf{A}+\mathbf{B})$$

$$=tr\left(\boldsymbol{\Sigma}^{-1}n\hat{\boldsymbol{\Sigma}}\right)$$

$$=ntr\left(\boldsymbol{\Sigma}^{-1}\hat{\boldsymbol{\Sigma}}\right) \qquad \text{since } tr(a\mathbf{A})=atr(\mathbf{A})$$

$$\therefore \frac{1}{n}\sum_{i=1}^{n}\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)'\boldsymbol{\Sigma}^{-1}\left(\mathbf{X}_i-\overline{\mathbf{X}}\right)=\frac{1}{n}\left[ntr\left(\boldsymbol{\Sigma}^{-1}\hat{\boldsymbol{\Sigma}}\right)\right]=tr\left(\boldsymbol{\Sigma}^{-1}\hat{\boldsymbol{\Sigma}}\right)$$