STA 312F2007 Solutions to Quiz 5

1.

$$Var(X) = Var(\xi + \delta) = Var(\xi) + Var(\delta) = \phi + \theta_\delta$$
$$Var(Y_1) = Var(\gamma_1\xi + \zeta_1) = \gamma_1^2 Var(\xi) + Var(\zeta_1) = \gamma_1^2\phi + \psi_1$$
$$Var(Y_2) = Var(\gamma_2\xi + \zeta_2) = \gamma_2^2 Var(\xi) + Var(\zeta_2) = \gamma_2^2\phi + \psi_2$$
$$Cov(X, Y_1) = Cov(\xi + \delta, \gamma_1\xi + \zeta_1) = \gamma_1 Var(\xi) = \gamma_1\phi$$
$$Cov(X, Y_2) = Cov(\xi + \delta, \gamma_2\xi + \zeta_2) = \gamma_2 Var(\xi) = \gamma_2\phi$$
$$Cov(Y_1, Y_2) = Cov(\gamma_1\xi + \zeta_1, \gamma_2\xi + \zeta_2) = \gamma_1\gamma_2 Var(\xi) = \gamma_1\gamma_2\phi$$

$$\mathbf{\Sigma} = \begin{pmatrix} \phi + \theta_\delta & \gamma_1\phi & \gamma_2\phi \\ & \gamma_1^2\phi + \psi_1 & \gamma_1\gamma_2\phi \\ & & \gamma_2^2\phi + \psi_2 \end{pmatrix}$$

2. $\theta = \{\phi, \theta_\delta, \gamma_1, \gamma_2, \psi_1, \psi_2\}$

3. Yes, the model is identified.

$$\begin{cases} \sigma_{11} = \phi + \theta_\delta & -(1) \\ \sigma_{12} = \gamma_1\phi & -(2) \\ \sigma_{13} = \gamma_2\phi & -(3) \\ \sigma_{22} = \gamma_1^2\phi + \psi_1 & -(4) \\ \sigma_{23} = \gamma_1\gamma_2\phi & -(5) \\ \sigma_{33} = \gamma_2^2\phi + \psi_2 & -(6) \end{cases}$$

$$\frac{(5)}{(2)} : \gamma_2 = \frac{\sigma_{23}}{\sigma_{12}}$$
$$(3) : \phi = \frac{\sigma_{13}}{\gamma_2}$$
$$(2) : \gamma_1 = \frac{\sigma_{12}}{\phi}$$
$$(1) : \theta_\delta = \sigma_{11} - \phi$$
$$(4) : \psi_1 = \sigma_{22} - \gamma_1^2\phi$$
$$(6) : \psi_2 = \sigma_{33} - \gamma_2^2\phi$$

One solution is obtained therefore the model is identified.