

### STA 312F2007 Solutions to Quiz 3

1. Note that  $E(\mathbf{AX}) = \left[ E\left(\sum_k A_{ik} X_{kj}\right) \right] = \left[ \sum_k A_{ik} E(X_{kj}) \right] = \mathbf{A}E(\mathbf{X})$

$$\begin{aligned} V(\mathbf{AX}) &= E(\mathbf{AX} - \boldsymbol{\mu}_{\mathbf{AX}})(\mathbf{AX} - \boldsymbol{\mu}_{\mathbf{AX}})' \\ &= E(\mathbf{AX} - \mathbf{A}\boldsymbol{\mu}_{\mathbf{X}})(\mathbf{AX} - \mathbf{A}\boldsymbol{\mu}_{\mathbf{X}})' \\ &= E[\mathbf{A}(\mathbf{X} - \boldsymbol{\mu}_{\mathbf{X}})][\mathbf{A}(\mathbf{X} - \boldsymbol{\mu}_{\mathbf{X}})]' \\ &= \mathbf{A}E(\mathbf{X} - \boldsymbol{\mu}_{\mathbf{X}})(\mathbf{X} - \boldsymbol{\mu}_{\mathbf{X}})' \mathbf{A}' \\ &= \mathbf{A}V(\mathbf{X})\mathbf{A}' = \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}' \end{aligned}$$

2. Note that  $E(\mathbf{Y})' = \left[ E(Y_{ij}) \right]' = \left[ E(Y_{ji}) \right] = E(\mathbf{Y}')$

$$\begin{aligned} C(\mathbf{X}, \mathbf{Y}) &= E(\mathbf{X} - \boldsymbol{\mu}_{\mathbf{X}})(\mathbf{Y} - \boldsymbol{\mu}_{\mathbf{Y}})' \\ &= E(\mathbf{X} - \boldsymbol{\mu}_{\mathbf{X}})(\mathbf{Y}' - \boldsymbol{\mu}'_{\mathbf{Y}}) \\ &= E(\mathbf{X}\mathbf{Y}' - \mathbf{X}\boldsymbol{\mu}'_{\mathbf{Y}} - \boldsymbol{\mu}_{\mathbf{X}}\mathbf{Y}' + \boldsymbol{\mu}_{\mathbf{X}}\boldsymbol{\mu}'_{\mathbf{Y}}) \\ &= E(\mathbf{X}\mathbf{Y}') - E(\mathbf{X})\boldsymbol{\mu}'_{\mathbf{Y}} - \boldsymbol{\mu}_{\mathbf{X}}E(\mathbf{Y}') + \boldsymbol{\mu}_{\mathbf{X}}\boldsymbol{\mu}'_{\mathbf{Y}} \\ &= E(\mathbf{X}\mathbf{Y}') - \boldsymbol{\mu}_{\mathbf{X}}\boldsymbol{\mu}'_{\mathbf{Y}} - \boldsymbol{\mu}_{\mathbf{X}}\boldsymbol{\mu}'_{\mathbf{Y}} + \boldsymbol{\mu}_{\mathbf{X}}\boldsymbol{\mu}'_{\mathbf{Y}} \\ &= E(\mathbf{X}\mathbf{Y}') - \boldsymbol{\mu}_{\mathbf{X}}\boldsymbol{\mu}'_{\mathbf{Y}} \end{aligned}$$