

STA 312F2007 Solutions to Quiz 2

1.

$$\begin{aligned} E(\xi_1) &= E\left(\sqrt{\frac{1+\phi}{2}}R_1 + \sqrt{\frac{1-\phi}{2}}R_2\right) = \sqrt{\frac{1+\phi}{2}}E(R_1) + \sqrt{\frac{1-\phi}{2}}E(R_2) \\ &= \sqrt{\frac{1+\phi}{2}}(0) + \sqrt{\frac{1-\phi}{2}}(0) = 0 \end{aligned}$$

2.

$$\begin{aligned} E(\xi_2) &= E\left(\sqrt{\frac{1+\phi}{2}}R_1 - \sqrt{\frac{1-\phi}{2}}R_2\right) = \sqrt{\frac{1+\phi}{2}}E(R_1) - \sqrt{\frac{1-\phi}{2}}E(R_2) \\ &= \sqrt{\frac{1+\phi}{2}}(0) - \sqrt{\frac{1-\phi}{2}}(0) = 0 \end{aligned}$$

3.

$$\begin{aligned} \text{Var}(\xi_1) &= \text{Var}\left(\sqrt{\frac{1+\phi}{2}}R_1 + \sqrt{\frac{1-\phi}{2}}R_2\right) = \frac{1+\phi}{2}\text{Var}(R_1) + \frac{1-\phi}{2}\text{Var}(R_2) + 0 \\ &= \frac{1+\phi}{2}(1) + \frac{1-\phi}{2}(1) = 1 \end{aligned}$$

4.

$$\begin{aligned} \text{Var}(\xi_2) &= \text{Var}\left(\sqrt{\frac{1+\phi}{2}}R_1 - \sqrt{\frac{1-\phi}{2}}R_2\right) = \frac{1+\phi}{2}\text{Var}(R_1) + \frac{1-\phi}{2}\text{Var}(R_2) + 0 \\ &= \frac{1+\phi}{2}(1) + \frac{1-\phi}{2}(1) = 1 \end{aligned}$$

5.

$$\begin{aligned} \text{Cov}(\xi_1, \xi_2) &= \text{Cov}\left(\sqrt{\frac{1+\phi}{2}}R_1 + \sqrt{\frac{1-\phi}{2}}R_2, \sqrt{\frac{1+\phi}{2}}R_1 - \sqrt{\frac{1-\phi}{2}}R_2\right) \\ &= \text{Var}\left(\sqrt{\frac{1+\phi}{2}}R_1\right) - \text{Var}\left(\sqrt{\frac{1-\phi}{2}}R_2\right) \\ &= \frac{1+\phi}{2}(1) - \frac{1-\phi}{2}(1) = \phi \end{aligned}$$

6.

$$\text{Corr}(\xi_1, \xi_2) = \frac{\text{Cov}(\xi_1, \xi_2)}{\text{SD}(\xi_1)\text{SD}(\xi_2)} = \frac{\phi}{\sqrt{1}\sqrt{1}} = \phi$$