STA 312F2007 Solutions to Quiz 1

1.

$$\begin{aligned} Cov(X+a, Y+b) &= E\left[(X+a-\mu_{X+a})(Y+b-\mu_{Y+b})\right] \\ &= E\left[(X+a-(\mu_X+a))(Y+b-(\mu_Y+b))\right] \\ &= E(XY) \end{aligned}$$

2. Bernoulli distribution with probability of success θ

a.

$p(x) = \theta^x (1-\theta)^{1-x}$ for $x = 0,1$ where $0 < \theta < 1$

$$L(\theta) = \prod_{i=1}^n \theta^{x_i} (1-\theta)^{1-x_i} = \theta^{\sum_{i=1}^n x_i} (1-\theta)^{n-\sum_{i=1}^n x_i}$$

$$l(\theta) = \log[L(\theta)] = \sum_{i=1}^n x_i \log(\theta) + \left(n - \sum_{i=1}^n x_i\right)\log(1-\theta)$$

$$l'(\theta) = \sum_{i=1}^n x_i \frac{1}{\theta} + \left(n - \sum_{i=1}^n x_i\right)\frac{1}{1-\theta}(-1) = \frac{\sum_{i=1}^n x_i}{\theta} - \frac{n - \sum_{i=1}^n x_i}{1-\theta}$$

Set $l'(\theta) = 0$

$$\Rightarrow \frac{\sum_{i=1}^n x_i}{\hat{\theta}} - \frac{n - \sum_{i=1}^n x_i}{1-\hat{\theta}} = 0$$

$$\Rightarrow \sum_{i=1}^n x_i (1-\hat{\theta}) - \left(n - \sum_{i=1}^n x_i\right)\hat{\theta} = 0$$

$$\Rightarrow \sum_{i=1}^n x_i - \left(\sum_{i=1}^n x_i + n - \sum_{i=1}^n x_i\right)\hat{\theta} = 0$$

$$\Rightarrow \sum_{i=1}^n x_i = n\hat{\theta}$$

$$\Rightarrow \hat{\theta} = \frac{\sum_{i=1}^n x_i}{n} = \bar{x}$$

b.

Data:

$n = 3 \qquad \sum_{i=1}^n x_i = 1$

$\hat{\theta} = \frac{1}{3}$