STA312F07 Quiz 9

1. In assignment 9, we have proved that the model with 3 instrumental variables is still identified. Now, let us add one more instrumental variable into model. The model then becomes:

$$Y_j = \gamma_j \xi + \zeta_j \text{ for } j = 1, \ldots, 5$$
$$X = \xi + \delta,$$

where δ, ξ, ζ_j for $j = 1, \ldots, 5$ are all independent, $Var(\xi) = \phi$, $Var(\zeta_j) = \psi_j$ for $j = 1, \ldots, 5$, $Var(\delta) = \theta_\delta$, all expected values are zero, and the regression coefficients γ_j for $j = 1, \ldots, 5$ are fixed constants. The model is presented for a single observation, and implicitly everything is independent and identically distributed, for $i = 1, \ldots, n$. Note that this is the model we used for the poverty data in assignment 9.

   (a) Writing the vector of observable data (for subject i) as $\mathbf{D} = (X, Y_1, Y_2, Y_3, Y_4, Y_5)'$, the variance-covariance matrix $\mathbf{\Sigma}$ has size 6×6. Show that $\sigma_{16} = \gamma_5 \phi$.

   (b) With the addition of one more instrumental variable introduces 5 more identifying equations

$$\sigma_{16} = \gamma_5 \phi$$
$$\sigma_{26} = \gamma_1 \gamma_5 \phi$$
$$\sigma_{36} = \gamma_2 \gamma_5 \phi$$
$$\sigma_{46} = \gamma_3 \gamma_5 \phi$$
$$\sigma_{66} = \gamma_5^2 \phi + \psi_5$$

   and 2 more unknowns γ_5 and ψ_5. Show that the additional unknowns can be solved using the additional identifying equations. Therefore, the model is identified.

2. Refer to your SAS output, test if GNP affects birth rate. State the null and alternative hypotheses, test statistic, decision rule (use $\alpha = 0.05$), decision and conclusion in plain English.

   You may find one of these numbers useful: $z_{0.05} = 1.645$, $z_{0.025} = 1.96$, $z_{0.005} = 2.576$.

3. Hand in the *log* file.

Total Marks = 10 points