STA 312F07 Quiz 3

1. (5 points) If the $p \times 1$ random vector $\mathbf{X}$ has variance-covariance matrix $\mathbf{\Sigma}$ and $\mathbf{A}$ is an $m \times p$ matrix of constants, prove that the variance-covariance matrix of $\mathbf{AX}$ is $\mathbf{A\Sigma A}'$. Start with the definition of a variance-covariance matrix:

$$V(\mathbf{Z}) = E\left((\mathbf{Z} - \boldsymbol{\mu}_z)(\mathbf{Z} - \boldsymbol{\mu}_z)'\right).$$

2. (5 points) Let $\mathbf{X}$ be a $p \times 1$ random vector with mean $\boldsymbol{\mu}_x$, and let $\mathbf{Y}$ be an $r \times 1$ random vector with mean $\boldsymbol{\mu}_y$. Defining $C(\mathbf{X}, \mathbf{Y})$ by the $p \times r$ matrix $C(\mathbf{X}, \mathbf{Y}) = E\Big((\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{Y} - \boldsymbol{\mu}_y)'\Big)$ Show $C(\mathbf{X}, \mathbf{Y}) = E(\mathbf{XY}') - \boldsymbol{\mu}_x \boldsymbol{\mu}_y'$.

Total Marks = 10 Points