STA 312f07 Assignment 7

Do this assignment in preparation for the quiz on Friday, Nov. 2nd. The questions are practice for the quiz, and are not to be handed in.

1. Let $Y_1, \ldots, Y_n$ be a random sample from a distribution with density $f(y) = \frac{1}{\theta} e^{-\frac{y}{\theta}}$ for $y > 0$, where the parameter $\theta > 0$. We are interested in testing $H_0 : \theta = \theta_0$.

   (a) What is Θ?

   (b) What is Θ_0?

   (c) Derive a general expression for the large-sample likelihood ratio statistic $G = -2 \log \frac{\mathcal{L}(\widehat{\widehat{\theta}})}{\mathcal{L}(\widehat{\theta})}$.

   (d) A sample of size $n = 100$ yields $\overline{Y} = 1.37$ and $S^2 = 1.42$. One of these quantities is unnecessary and just provided to irritate you. Well, actually it's a mild substitute for reality, which always provides you with a huge pile of information you don't need. Anyway, we want to test $H_0 : \theta = 1$. You can do this with a calculator. When I did it a long time ago I got $G = 11.038$.

   (e) At $\alpha = 0.05$, the critical value of chisquare with one degree of freedom is 3.841459. Do you reject H_0? Answer Yes or No.

2. The label on the peanut butter jar says peanuts, partially hydrogenated peanut oil, salt and sugar. But we all know there is other stuff in there too. In the United States, the Food and Drug administration requires that a shipment of peanut butter be rejected if it contains an average of more than 8 rat hairs per pound (well, I'm not sure if it's exactly 8, but let's pretend). There is very good reason to assume that the number of rat hairs per pound has a Poisson distribution with mean λ (take STA347). We will test $H_0 : \lambda = \lambda_0$.

   (a) What is Θ?

   (b) What is Θ_0?

   (c) Derive a general expression for the large-sample likelihood ratio statistic.

   (d) We sample 100 1-pound jars, and observe a sample mean of $\overline{Y} = 8.57$. Should we reject the shipment? We want to test $H_0 : \lambda = 8$. What is the value of G? You can do this with a calculator. When I did it a long time ago I got $G = 3.97$.

   (e) Do you reject H_0 at $\alpha = 0.05$? Answer Yes or No.

   (f) Do you reject the shipment of peanut butter? Answer Yes or No.

3. The normal distribution has density

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\}.$$

   (a) Using the MLE of $\theta = (\mu, \sigma^2)$ without proof, derive the large-sample likelihood ratio test of $H_0 : \sigma^2 = \sigma_0^2$ for data from a single normal random sample.

   (b) Use the following data to test $H_0 : \sigma^2 = 2$

```
0.05850114 -0.3579544 0.2937383 -1.637927 0.8620997
1.625934 0.3769822 1.692480 -0.8440954 3.876633
-1.654357 3.665209 1.254225 1.411258 -2.716365
3.547531 -0.2264552 -2.363213 -0.06207292 0.7077028
1.481790 -0.8427953 -3.342643 3.544554 -2.764204
```

      i. What is the value of G? Your answer is a single number.

      ii. Do you reject H_0? Answer Yes or No.

4. Let $\mathbf{X}_1, \ldots, \mathbf{X}_n$ be a random sample from a k-variable multivariate normal population with mean $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$. Derive the large-sample likelihood ratio test G for testing whether the components of the random vectors $\mathbf{X}_i$ are independent. That is, we want to test whether $\boldsymbol{\Sigma}$ is diagonal. Specifically, show that

$$G = n(\sum_{j=1}^{k} \log \widehat{\sigma}_j^2 - \log|\widehat{\boldsymbol{\Sigma}}|)$$

This is challenging; here is some help.

- Remember that normal random variables have zero covariance if and only if they are independent, and you know the MLEs for a single sample. This means you can just write down $\widehat{\widehat{\boldsymbol{\Sigma}}}$.
- The determinant of a diagonal matrix is the product of the main diagonal.
- Feel free to use the expression for $-2\log\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ from the last assignment.
- $tr(\widehat{\widehat{\boldsymbol{\Sigma}}}^{-1}\widehat{\boldsymbol{\Sigma}})$ simplifies in a big way; examine it carefully.