# STA 312F2007 Assignment 1

Do this review assignment in preparation for the quiz on Friday, Sept. 21st. The problems are practice for the quiz, and are not to be handed in.

1. Let the random vector $\mathbf{X} = (X_1, \ldots, X_n)'$ have density $f(\mathbf{x})$. For a general function $g$, use the rule $E[g(\mathbf{X})] = \int \cdots \int g(\mathbf{x}) f(\mathbf{x}) d\mathbf{x}$ as if it were a definition. Prove
   (a) $E[a] = a$, where $a$ is a constant.
   (b) $E[aX] = aE[X]$, where $a$ is a constant.
   (c) $E[\sum_{i=1}^n X_i] = \sum_{i=1}^n E[X_i]$
   (d) If $X$ and $Y$ are independent, $E[XY] = E[X]E[Y]$. Draw an arrow to the place in your answer where you use independence, and write "This is where I use independence."

   In all the remaining questions about variances and covariances, you should just use the linear properties of expectation that you have proved in Question 1, and avoid using integrals.

2. Denoting $E[X]$ by $\mu_x$, define the variance $Var(X) = E[(X - \mu_x)^2]$. Show that $Var(X) = E[X^2] - \mu_x^2$.

3. Define the covariance of $X$ and $Y$ by $Cov[X, Y] = E[(X - \mu_x)(Y - \mu_y)]$. Show that $Cov[X, Y] = E[XY] - \mu_x \mu_y$.

4. In the following, $X$ and $X_1, \ldots, X_n$ are random variables, while $a$, $b$ and $a_1, \ldots, a_n$ are fixed constants. For each pair of statements below, one is true and one is false (that is, not true in general). State which one is true, and prove it. Zero marks if you prove both statements are true, even if one of the proofs is correct.
   (a) $Var(aX) = aVar(X)$ or $Var(aX) = a^2 Var(X)$
   (b) $Var(aX + b) = a^2 Var(X) + b^2$ or $Var(aX + b) = a^2 Var(X)$
   (c) $Var(a) = 0$ or $Var(a) = a^2$
   (d) $Cov(X + a, Y + b) = Cov(X, Y) + ab$ or $Cov(X + a, Y + b) = Cov(X, Y)$
   (e) $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$ or $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$

5. For each of the following distributions, derive a general expression for the Maximum Likelihood Estimator (MLE). You don't have to do the second derivative test. Then use the data to calculate a numerical estimate.
   (a) $p(x) = \theta(1 - \theta)^x$ for $x = 0, 1, \ldots$, where $0 < \theta < 1$. Data: `4, 0, 1, 0, 1, 3, 2, 16, 3, 0, 4, 3, 6, 16, 0, 0, 1, 1, 6, 10`. Answer: 0.2061856
   (b) $f(x) = \frac{\alpha}{x^{\alpha+1}}$ for $x > 1$, where $\alpha > 0$. Data: `1.37, 2.89, 1.52, 1.77, 1.04, 2.71, 1.19, 1.13, 15.66, 1.43` Answer: 1.469102
   (c) $f(x) = \frac{\tau}{\sqrt{2\pi}} e^{-\frac{\tau^2 x^2}{2}}$, for $x$ real, where $\tau > 0$. Data: `1.45, 0.47, -3.33, 0.82, -1.59, -0.37, -1.56, -0.20` Answer: 0.6451059
   (d) $f(x) = \frac{1}{\theta} e^{-x/\theta}$ for $x > 0$, where $\theta > 0$. Data: `0.28, 1.72, 0.08, 1.22, 1.86, 0.62, 2.44, 2.48, 2.96` Answer: 1.517778