One-sample Z-test (single mean)

Model Data are a random sample from a large population with mean μ and standard deviation σ

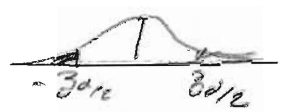
H_0 $\mu = \mu_0$

H_a $\mu \neq \mu_0$. Rarely, $\mu < \mu_0$ or $\mu > \mu_0$

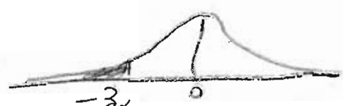
Test statistic: $Z = \dfrac{\bar{X} - \mu_0}{s/\sqrt{n}}$

Rejection region

a) (2-sided) $H_a: \mu \neq \mu_0$ $|Z| > z_{\alpha/2}$



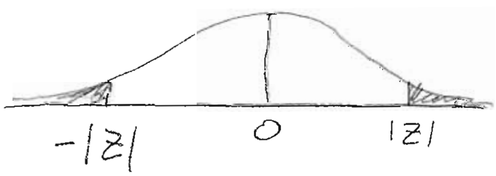
b) (1-sided) $H_a: \mu < \mu_0$ $Z < -z_{\alpha}$



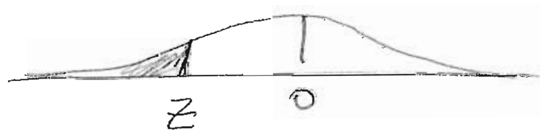
c) (1-sided) $H_a: \mu > \mu_0$ $Z > z_{\alpha}$

P-value

(a) $H_a: \mu \neq \mu_0$

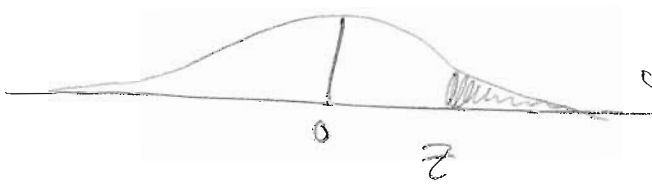


(b) $H_a: \mu < \mu_0$

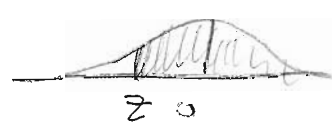


or

(c) H_a $\mu > \mu_0$



or



One-sample t-test

Model: Data are a random sample from a normal population with mean μ & standard deviation σ.

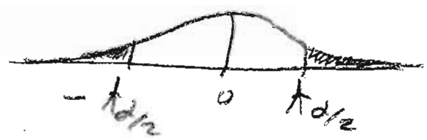
H_0 : $\mu = \mu_0$

H_a $\mu \neq \mu_0$. Rarely, $\mu < \mu_0$ or $\mu > \mu_0$

Test statistic: $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$

Rejection region

a) $H_a : \mu \neq \mu_0 : |t| > t_{\alpha/2}$



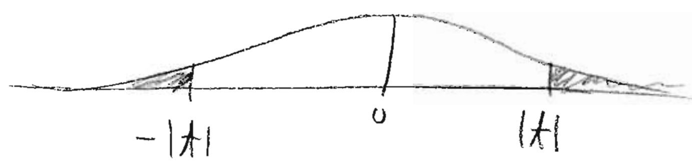
b) $H_a : \mu < \mu_0 : t < -t_\alpha$

c) $H_a : \mu > \mu_0 : t > t_\alpha$

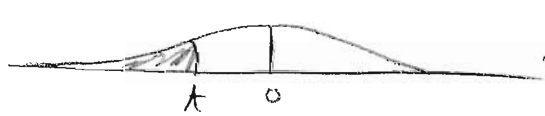
All critical values are based on $n-1$ degrees of freedom

P-value

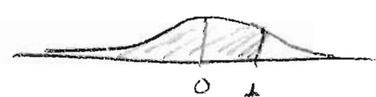
a) $H_a : \mu \neq \mu_0$



b) $H_a : \mu < \mu_0$

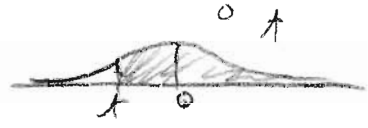


or



c) $H_a : \mu > \mu_0$

or



Z-test for a single proportion

Model: Data are a random sample from a large binary population with $P(X=1)=p$ and $P(X=0)=1-p$

$H_0: p = p_0$

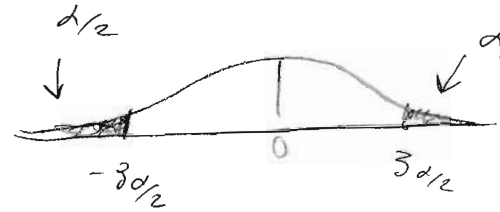
$H_a: \quad p \neq p_0 \qquad p > p_0 \qquad p < p_0$

Test statistic $\quad Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} = \dfrac{\sqrt{n}(\hat{p} - p_0)}{\sqrt{p_0(1-p_0)}}$

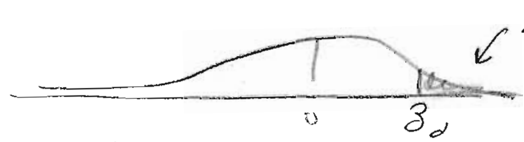
Significance Level α

Rejection Region

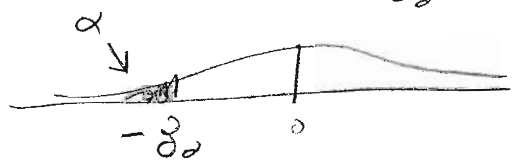
For $H_a: p \neq p_0 \qquad |Z| > z_{\alpha/2}$



For $H_a: p > p_0 \qquad Z > z_\alpha$

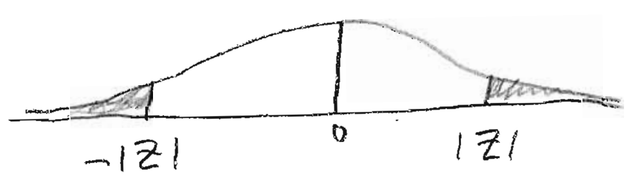


For $H_a: p < p_0 \qquad Z < -z_\alpha$

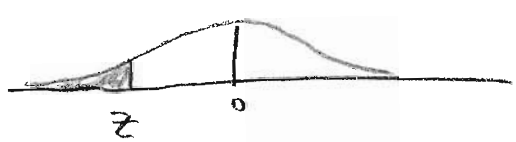


P-value

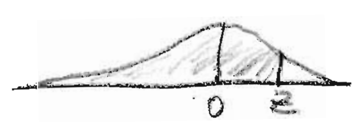
(a) $H_a: \mu \neq \mu_0$



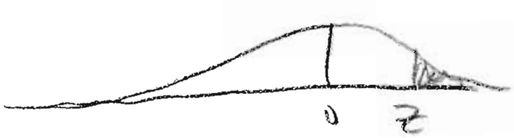
(b) $H_a: \mu < \mu_0$



or



(c) $H_a: \mu > \mu_0$



or

Z-test for two means

<u>Model</u> Data are independent random samples from large populations with means μ_1 & μ_2, and standard deviations σ_1 & σ_2

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$. Rarely, $\mu_1 < \mu_2$ or $\mu_1 > \mu_2$

<u>Test statistic</u>

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

<u>Rejection region</u>

For $H_a: \mu_1 \neq \mu_2$ $\quad |Z| > z_{\alpha/2}$

For $H_a: \mu_1 > \mu_2$ $\quad Z > z_\alpha$

For $H_a: \mu_1 < \mu_2$ $\quad Z < -z_\alpha$

t-test for two means

(Also called independent t-test or two-sample t-test)

Model: Data are independent random samples from normal populations with means μ_1 & μ_2, and the same variance σ^2.

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \neq \mu_2$. Rarely, $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$

Test Statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}, \text{ where } s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

Rejection region

For $H_a: \mu_1 \neq \mu_2$ $\quad |t| > t_{\alpha/2}$

For H_a $\mu_1 > \mu_2$ $\quad t > t_\alpha$

For H_a $\mu_1 < \mu_2$ $\quad t < -t_\alpha$

All critical values are based on $n_1 + n_2 - 2$ degrees of freedom.

Z - test for 2 proportions

Model Data are independent random samples from large binary populations with proportions of 1's $p_1 \neq p_2$.

$H_0: p_1 = p_2$

$H_a \; p_1 \neq p_2$, Rarely, $p_1 > p_2$ or $p_1 < p_2$

Test statistic

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \quad \text{where} \quad \hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1+n_2}$$

Rejection Region

For $H_a: p_1 \neq p_2$ $\quad |Z| > z_{\alpha/2}$

For $H_a: p_1 > p_2$ $\quad Z > z_\alpha$

For $H_a: p_1 < p_2$ $\quad Z < -z_\alpha$