Theorem 1 *Suppose that $X_1, \ldots, X_n$ is a random sample from a normal distribution with an unknown value of the mean M and an unknown value of the precision R. Suppose also that the prior joint distribution of M and R is as follows: The conditional distribution of M when $R = r$ $(r > 0)$ is a normal distribution with mean μ and precision τr such that $-\infty < \mu < \infty$ and $\tau > 0$, and the marginal distribution of R is a gamma distribution with parameters α and β such that $\alpha > 0$ and $\beta > 0$. Then the posterior joint distribution of M and R when $X_i = x_i$ $(i = 1, \ldots, n)$ is as follows: The conditional distribution of M when $R = r$ is a normal distribution with mean μ' and precision $(\tau + n)r$, where*

$$\mu' = \frac{\tau\mu + n\bar{x}}{\tau + n}, \tag{1}$$

and the marginal distribution of R is a gamma distribution with parameters $\alpha + (n/2)$ and β', where

$$\beta' = \beta + \frac{1}{2}\sum_{i=1}^{n}(x_i - \bar{x})^2 + \frac{\tau n(\bar{x} - \mu)^2}{2(\tau + n)}. \tag{2}$$