STA 2101/442 Assignment Ten[1]

For this assignment, there is a computer part to each question. Use R, and bring your printouts to the quiz. **Your printouts should show *all* R input and output, and *only* R input and output**. Do not write anything on your printouts except your name and student number.

1. I know this is pretty gruesome, but the data are real. An experiment in dentistry seeks to test the effectiveness of a drug (HEBP) that is supposed to help dental implants become more firmly attached to the jaw bone. This is an initial test on animals. False teeth were implanted into the leg bones of rabbits, and the rabbits were randomly assigned to receive either the drug or a saline solution (placebo). Technicians administering the drug were blind to experimental condition.

   Rabbits were also randomly assigned to be "sacrificed" after either 3, 6, 9 or 12 days. At that time, the implants were pulled out of the bone by a machine that measures force in newtons and stiffness in newtons/mm. For both of these measurements, higher values indicate more healing. A measure of "pre-load stiffness" in newtons/mm is also available for each animal. This may be another indicator of how firmly the false tooth was implanted into the bone, but it might even be a covariate. Nobody can seem to remember what "preload" means, so we'll ignore this variable for now.

   The explanatory variables are Time and Drug. The response variable is Force required to pull out the tooth. There is more than one reasonable way to do this analysis, but just to keep us together please treat Time as a categorical variable.

   The data are available in the file `bunnies.data.txt`. The variables are

   - Identification code
   - Time (3,6,9,12 days of healing)
   - Drug (1=HEBP, 0=saline solution)
   - Stiffness in newtons/mm
   - Force in newtons
   - Preload stiffness in newtons/mm

   (a) Use `table` to find out how many rabbits are in each experimental condition.

   (b) Carry out the standard tests of main effects and interactions. Be prepared to answer the following questions about each test.

      i. What is the value of the test statistic? The answer is a number from your printout.
      ii. What proportion of the remaining variation is explained? You can use `R`, or just be ready to do it with a calculator.
      iii. What is the p-value? The answer is a number from your printout.
      iv. Do you reject the null hypothesis at the 0.05 level? Yes or No.
      v. What, if anything, do you conclude? This is not the place for statistical jargon. "What do you conclude" means say something about the drug, healing, time – something like that.

   (c) I know this is a bit redundant with the preceding question, but *averaging across time, did the drug help the teeth become more firmly attached to the bone?* If the results justify an answer, then answer Yes or No.

   (d) Make a table with a row for each treatment combination. Make columns showing the dummy variables for effect coding.

   (e) Give $E[Y|\mathbf{X} = \mathbf{x}]$ for a regression model with both main effects and the interaction. Use your variable names from the preceding question.

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(f) In terms of the β values of your regression model, give the null hypothesis you would test in order to answer each of the following questions.

i. Averaging across time periods, is there a difference between the drug and placebo in mean force required to extract the tooth?

ii. Averaging across drug and placebo, is does elapsed time affect the mean force required to extract the tooth?

iii. Does the effect of the drug depend upon elapsed time?

(g) Now please return to R. Doing it the easiest way you can, conduct tests to answer the following questions. Just do regular one-at-a-time (custom) tests. Don't bother with any Bonferroni correction this time. Just consider one response variable: Force. As usual, we are guided by the $\alpha = 0.05$ significance level.

i. Are the marginal means different at 3 and 6 days?

ii. Are the marginal means different at 6 and 9 days?

iii. Are the marginal means different at 9 and 12 days?

iv. Is there a difference between Drug and Placebo just at 3 days?

v. Is there a difference between Drug and Placebo just at 6 days?

vi. Is there a difference between Drug and Placebo just at 9 days?

vii. Is there a difference between Drug and Placebo just at 12 days?

viii. Be able to answer questions like these for each test:

A. What is the value of the test statistic? The answer is a number from your printout.

B. What proportion of the remaining variation is explained? You don't have to do all these calculations in advance; just be ready to do them with a calculator.

C. What is the p-value? The answer is a number from your printout.

D. Do you reject the null hypothesis at the 0.05 level? Yes or No.

E. What, if anything, do you conclude? This is not the place for statistical jargon. "What do you conclude" means say something about the drug, healing, time – something like that.

2. Let $\mathbf{w} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma}$ is symmetric and positive definite.

(a) Show that $\mathbf{w}^\top \boldsymbol{\Sigma}^{-1} \mathbf{w}$ has a non-central chi-squared distribution with degrees of freedom p and non-centrality parameter $\lambda = \boldsymbol{\mu}^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}$.

(b) Use what you have just proved to obtain the formula for the non-centrality parameter for the general linear F-test; see the formula sheet. I think this is cleaner than the way it was done in lecture.

3. For this question, please feel free to use my code from the power lecture. Remember the rotten potato example of factorial ANOVA? It was a 2×3 design with nine cases per treatment combination. Suppose the true value of $\sigma^2 = 16$, and the true expected values are as follows:

	Bact1	Bact2	Bact3
Cool	4	4	12
Warm	5	5	14

   (a) For $n = 54$, what is the power to detect the main effect of temperature?

   (b) Still maintaining equal sample sizes, what minimum total sample size is required so that the test of the main effect for temperature will have a power of at least 0.8?

   (c) For $n = 54$, what is the power to detect the main effect of bacteria type?

   (d) Still maintaining equal sample sizes, what minimum total sample size is required so that the test of the main effect for bacteria type will have a power of at least 0.8?

   (e) For $n = 54$, what is the power to detect the interaction of temperature and bacteria type?

   (f) Still maintaining equal sample sizes, what minimum total sample size is required so that the test of the interaction will have a power of at least 0.8? Please use R and bring your printout to the quiz.

4. Let $Y_1, \ldots, Y_n$ be independent and identically distributed $N(\mu, \sigma^2)$ random variables. We are interested in the power of a test of $H_0 : \mu = \mu_0$ against the alternative that $\mu \neq 0$. The easy way out is to use a regression model with an intercept but no explanatory variables, so that the null hypothesis is a statement about the intercept.

   (a) Calculate the non-centrality parameter λ and simplify.

   (b) What is "effect size" for this problem?

   (c) Suppose $\frac{|\mu-\mu_0|}{\sigma} = \frac{1}{3}$. What is the minimum sample size required for a power of 0.80? The answer is a number.

Please bring your printouts Question 1, 3 and 4 to the quiz. **Your printouts should show *all* R input and output, and *only* R input and output**. Do not write anything on your printouts except your name and student number.
