STA 2101/442 Assignment Eleven[1]

1. Let $\mathbf{Y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Show that $W = \mathbf{Y}^\top \boldsymbol{\Sigma}^{-1} \mathbf{Y}$ has a non-central χ^2 distribution. What are the degrees of freedom? What is the non-centrality parameter λ? Use the formula sheet. The lecture slides will help with this question.

2. Recall the "Wald-like" tests based on the Central Limit Theorem:

$$W_n = n \left(\mathbf{L}\overline{\mathbf{X}}_n - \mathbf{h}\right)^\top \left(\mathbf{L}\widehat{\boldsymbol{\Sigma}}_n \mathbf{L}^\top\right)^{-1} \left(\mathbf{L}\overline{\mathbf{X}}_n - \mathbf{h}\right)$$

   When the null hypothesis is false, the test statistic has an approximate non-central chi-squared distribution. Based on your answer to Question 1, what is the non-centrality parameter?

3. Remember the rotten potato example of factorial ANOVA. It was a 2×3 design with nine cases per treatment combination. Suppose the true value of $\sigma^2 = 16$, and the true expected values are as follows:

	Bact1	Bact2	Bact3
Cool	4	4	12
Warm	5	5	14

   (a) For $n = 54$, what is the power to detect the main effect of temperature?

   (b) Still maintaining equal sample sizes, what minimum total sample size is required so that the test of the main effect for temperature will have a power of at least 0.8?

   (c) For $n = 54$, what is the power to detect the main effect of bacteria type?

   (d) Still maintaining equal sample sizes, what minimum total sample size is required so that the test of the main effect for bacteria type will have a power of at least 0.8?

   (e) For $n = 54$, what is the power to detect the interaction of temperature and bacteria type?

   (f) Still maintaining equal sample sizes, what minimum total sample size is required so that the test of the interaction will have a power of at least 0.8?

   (g) For $n = 54$, what is the power to detect an effect of bacteria type at cool temperatures?

4. Again for the rotten potato experiment, what sample size is required to reject the null hypothesis of no interaction, if the interaction explains 10% of the remaining sample variation in mean rot?

Please use R and bring your printout to the quiz. You may use my R functions if you wish. There are links from the course home page.

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