

STA 442/2101 f2013 Quiz 9

1. For the usual fixed effects multiple regression model (see formula sheet), let $\mathbf{Z} = \mathbf{X}'\mathbf{e}$.

(a) (2 points) Using (not proving) the fact that $V(\mathbf{e}) = \sigma^2(\mathbf{I}_n - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}')$, find $V(\mathbf{Z})$. Show your work.

$$\begin{aligned} V(\mathbf{Z}) &= \sigma^2 \mathbf{X}' (\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}') \mathbf{X} \\ &= \sigma^2 (\mathbf{X}'\mathbf{X} - \underbrace{\mathbf{X}'\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{X}}_{\mathbf{I}}) \\ &= \sigma^2 (\mathbf{X}'\mathbf{X} - \mathbf{X}'\mathbf{X}) = \mathbf{0} \end{aligned}$$

(b) (3 points) Recall that an $n \times n$ matrix \mathbf{A} is said to be *positive definite* if $\mathbf{v}'\mathbf{A}\mathbf{v} > 0$ for all non-zero vectors $\mathbf{v} \in \mathbb{R}^n$. How does your result above show that the variance-covariance matrix of \mathbf{e} is not positive definite?

Each column of \mathbf{X} is a non-zero vector $\mathbf{v} \in \mathbb{R}^n$ with $\mathbf{v}' \sigma^2 (\mathbf{I} - \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}') \mathbf{v} = 0$.

2. Based on your regression analysis of the furnace data, please answer the questions below by writing numbers from your printout. In case of doubt, base your answers on the full (unrestricted) model.

- (a) (1 Point) Controlling for type of chimney liner and chimney height, is age of house related to energy consumption?

Test Statistic (F or t)	p -value
$t = 0.08$	0.9374

- (b) (1 Point) Taking chimney height and age of house into account, is type of chimney liner related to energy consumption?

Test Statistic (F or t)	p -value
$F = 5.28$	0.0071

- (c) (1 Point) Allowing for age of house and chimney height, ^s is there an average difference in energy consumption between houses with metal chimney liners and houses with unlined chimneys?

Test Statistic (F or t)	p -value
$t = -3.23$	0.0088

- (d) (2 Points) In plain, non-statistical language, what do you conclude from that last test? Be guided by the $\alpha = 0.05$ significance level, but do not mention it.

Allowing for age of house and chimney height, houses with metal chimney liners tend to use less energy than houses with unlined chimneys.

Please attach your log file and your list file. Make sure your name appears on both files.