Name Jerry

Student Number

STA 442/2101 f2013 Quiz 6

1. Consider the usual fixed effects multiple regression model in scalar form:

$$Y_i = \beta_0 + \beta_1 x_{i,1} + \cdots + \beta_{p-1} x_{i,p-1} + \epsilon_i,$$

where the $x_{i,j}$ quantities are fixed observable constants, and the ϵ_i are unobservable random variables with expected value zero and variance σ^2. As you saw in homework, both the maximum likelihood and least squares estimates of the β_j are the quantities obtained by minimizing

$$Q(\boldsymbol{\beta}) = \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 x_{i,1} - \cdots - \beta_k x_{i,p-1})^2.$$

(a) (2 points) Differentiate $Q(\boldsymbol{\beta})$ with respect to β_0 and set the derivative to zero, obtaining the first *normal equation.*

$$\frac{\partial Q}{\partial \beta_0} = 2 \sum_{i=1}^{n} (Y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_{p-1} x_{i,p-1}) \overset{set}{=} 0$$

$$\Rightarrow \sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} (\beta_0 + \beta_1 x_{i1} + \cdots + \beta_{p-1} x_{i,p-1})$$

$$= n\beta_0 + \beta_1 \sum_{i=1}^{n} x_{i1} + \cdots + \beta_{p-1} \sum_{i=1}^{n} x_{i,p-1}$$

(b) (3 points) Noting that the quantities $\widehat{\beta}_0, \ldots, \widehat{\beta}_{p-1}$ must satisfy the first normal equation and defining "predicted" Y_i as $\widehat{Y}_i = \widehat{\beta}_0 + \widehat{\beta}_1 x_{i,1} + \cdots + \widehat{\beta}_{p-1} x_{i,p-1}$, show that $\sum_{i=1}^n \widehat{Y}_i = \sum_{i=1}^n Y_i$.

From (a),

$$\sum_{i=1}^n Y_i = \sum_{i=1}^n (\hat{\beta}_0 + \hat{\beta}_1 x_{i,1} + \cdots + \hat{\beta}_{p-1} x_{i,p-1}) = \sum_{i=1}^n \hat{Y}_i$$

2. (5 points) For the `normal` data of homework Problem One, you obtained the estimated asymptotic covariance matrix of $(\overline{Y}, \widehat{\sigma}^2)$ by finding the MLE numerically and then inverting the Hessian. Copy your answer (a 2×2 matrix of numbers) into the space below, and *attach the part of your printout that shows the calculation.* **Circle the matrix on your printout.** Do *not* turn in any unnecessary pages of computer output, but make sure you include the code that defined the minus log likelihood function and generated the Hessian.

```
R version 2.15.1 (2012-06-22) -- "Roasted Marshmallows"
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[Workspace restored from /Users/brunner/.RData]
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> rm(list=ls())
> y = scan("http://www.utstat.toronto.edu/~brunner/appliedf13/code_n_data/hw/normal.data")
Read 100 items
> n = length(y); ybar = mean(y) ; sigma2hat = (n-1)/n * var(y)
> n; ybar; sigma2hat
[1] 100
[1] 98.33
[1] 185.6211
> varhatmean =  sigma2hat/n; varhatmean
[1] 1.856211
>
> # Asyptotic covariance matrix based on hand calculated Fisher Information.
> AV1 = rbind(c(varhatmean,0),
+             c(0,2*sigma2hat^2/n)  ); AV1
         [,1]     [,2]
[1,] 1.856211   0.0000
[2,] 0.000000 689.1039
>
> # Asyptotic covariance matrix based on numerical search
> mll = function(theta,datta) # Minus LL for normal
+     {
+     mu = theta[1]; sigma2 = theta[2]
+     mll = -sum(dnorm(datta,mu,sqrt(sigma2),log=T))
+     mll
+     } # End of function mll
> search2 = nlm(mll,p=c(ybar,sigma2hat),hessian=T,datta=y)
> AV2 = solve(search2$hessian); AV2  # $
          [,1]        [,2]
[1,] 1.8562115   0.0182575
[2,] 0.0182575 689.3796758
>
>
```