Name Jerry

Student Number

STA 442/2101 f2013 Quiz 5

1. (6 point) The likelihood function for a multinomial can be written $L(\boldsymbol{\theta}) = \theta_1^{n_1}\theta_2^{n_2}\cdots\theta_c^{n_c}$, and the large-sample likelihood ratio test statistic is $G^2 = -2\log\left(\frac{\max_{\theta\in\Theta_0} L(\theta)}{\max_{\theta\in\Theta} L(\theta)}\right)$. For a random sample from a Multinomial$(1,\boldsymbol{\theta})$ distribution, show that the likelihood ratio test statistic can be written as

$$G^2 = 2n\sum_{j=1}^{c}\overline{Y}_j \log\left(\frac{\overline{Y}_j}{\widehat{\theta}_j}\right),$$

where $\widehat{\theta}_j$ is the *restricted* maximum likelihood estimate of θ_j. That is, it's the MLE under H_0. You may use without proof the fact that the unrestricted MLE is $\overline{Y}_j$.

$$G^2 = -2\log\frac{L(\hat{\underset{\sim}{\theta}})}{L(\overline{\underset{\sim}{Y}})} = -2\log\frac{\prod_{j=1}^{c}\hat{\theta}_j^{n_j}}{\prod_{j=1}^{c}\overline{Y}_j^{n_j}}$$

$$= -2\log\prod_{j=1}^{c}\left(\frac{\hat{\theta}_j}{\overline{Y}_j}\right)^{n_j} = -2\sum_{j=1}^{c} n_j\log\left(\frac{\hat{\theta}_j}{\overline{Y}_j}\right)$$

$$= 2\sum_{j=1}^{c} n_j\log\left(\frac{\overline{Y}_j}{\hat{\theta}_j}\right) = 2n\sum_{j=1}^{c}\frac{n_j}{n}\log\left(\frac{\overline{Y}_j}{\hat{\theta}_j}\right)$$

$$= 2n\sum_{j=1}^{c}\overline{Y}_j\log\left(\frac{\overline{Y}_j}{\hat{\theta}_j}\right)$$

2. (4 points) In homework problems 1b and 2, you estimated the parameters of a beta distribution, and tested $H_0 : \alpha = \beta$. Give the following numbers from your printouts. **You do *not* need to turn in the printouts.**

(a) The value of $\widehat{\alpha}$. The answer is a number.

13.97

(b) The value of $\widehat{\beta}$. The answer is a number.

27.28

(c) The value of the likelihood ratio test statistic G^2. The answer is a number.

42.13

(d) The p-value. The answer is a number. R's "scientific" notation is okay.

8.53e-11, or

0.0000000000853 2719

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R version 2.15.1 (2012-06-22) -- "Roasted Marshmallows"
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> rm(list=ls())
> x <- scan("http://www.utstat.toronto.edu/~brunner/appliedf13/code_n_data/hw/beta.data")
Read 50 items
>
> bll = function(ab,datta) # - Loglike of beta
+     { bll = -sum(dbeta(datta,ab[1],ab[2],log=T)); bll }
>
> # nlm(bll,c(1,1),datta=x) # Works, with warnings.
>
> fit1 <- nlminb(c(1,1),objective=bll,lower=c(0,0),datta=x); fit1
$par
[1] 13.96757 27.27780

$objective
[1] -60.26451

$convergence
[1] 0

$iterations
[1] 19

$evaluations
function gradient
      20       46

$message
[1] "relative convergence (4)"

>
> bll0 = function(theta,datta) # - Loglike of beta under H0: alpha=beta
+     { bll0 = -sum(dbeta(datta,theta,theta,log=T)); bll0 }
>
> fit0 = nlminb(1,objective=bll0,lower=0,datta=x); fit0
$par
[1] 3.796628
```

```
$objective
[1] -18.13277

$convergence
[1] 0

$iterations
[1] 9

$evaluations
function gradient
      10       12

$message
[1] "relative convergence (4)"

>
> # nlm(bll0,p=1,datta=x) # Works with warnings
>
> # G^2 is twice difference between (minus) log likelihoods)
> G2 = fit0$objective-fit1$objective; G2
[1] 42.13173
>
> df=1
> pval = 1-pchisq(G2,df); pval
[1] 8.532719e-11
>
>
```