Name Jerry

Student Number ____________

STA 442/2101 f2012 Quiz 1

1. (2 points) Let $\mathbf{z} = (z_1, \ldots, z_n)'$ be a vector of real constants. Show $\mathbf{z}'\mathbf{z} \geq 0$.

$$z'z = \sum_{i=1}^{n} z_i^2 \geq 0$$

2. (4 points) Recall the definition of linear independence. The columns of $\mathbf{A}$ are said to be *linearly independent* if the only column vector $\mathbf{v}$ with $\mathbf{Av} = \mathbf{0}$ is $\mathbf{v} = \mathbf{0}$. That is, $\mathbf{Av} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

   Let $\mathbf{X}$ be an $n \times p$ matrix of constants. Show that if the columns of $\mathbf{X}$ are linearly independent, then the columns of $\mathbf{X}'\mathbf{X}$ are also linearly independent.

$X'Xv = 0 \Rightarrow v'X'Xv = (Xv)'Xv =$

$= z'z = 0$ with $z = Xv$, so $Xv = 0$

So $v = 0$ by linear independence of the cols of X.

They don't have to write quite so much.

3. (4 points) Let $Y_1, \ldots, Y_n$ be a random sample from a normal distribution with mean μ and variance σ^2, so that $T = \frac{\sqrt{n}(\overline{Y}-\mu)}{S} \sim t(n-1)$. You may use this fact without proof.

(a) Derive a $(1-\alpha)100\%$ confidence interval for μ. "Derive" means show all the high school algebra. Use the symbol $t_{\alpha/2}$ for the number satisfying $Pr(T > t_{\alpha/2}) = \alpha/2$.

$$1-\alpha = P(-t_{\alpha/2} < T < t_{\alpha/2})$$

$$= P\left(-t_{\alpha/2} < \frac{\sqrt{n}(\overline{Y}-\mu)}{S} < t_{\alpha/2}\right)$$

$$= P\left(-t_{\alpha/2}\frac{S}{\sqrt{n}} < \overline{Y}-\mu < t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$

$$= P\left(-\overline{Y}-t_{\alpha/2}\frac{S}{\sqrt{n}} < -\mu < -\overline{Y}+t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$

$$= P\left(\overline{Y}+t_{\alpha/2}\frac{S}{n} > \mu > \overline{Y}-t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$

$$= P\left(\overline{Y}-t_{\alpha/2}\frac{S}{\sqrt{n}} < \mu < \overline{Y}+t_{\alpha/2}\frac{S}{\sqrt{n}}\right)$$ This step is optional

(b) A random sample with $n = 23$ yields $\overline{Y} = 2.57$ and a sample variance of $S^2 = 5.85$. Calculate the test statistic for testing $H_0 : \mu = 2$. Show some work. Your answer is a number.

$$T = \frac{\sqrt{23}\,(2.57-2)}{\sqrt{5.85}} = 1.13$$