December 05, 2013

TEMP

	Low	Med	High
Low	μ_1	μ_2	μ_3
High	μ_4	μ_5	μ_6

$\mu_1-\mu_4=(\mu_2-\mu_5)=\mu_3-\mu_6$

$\mu_1+\mu_4=\mu_2+\mu_5=\mu_3+\mu_6$

$$L=\begin{pmatrix}1 & -1 & 0 & 1 & -1 & 0\\ 0 & 1 & -1 & 0 & 1 & -1\end{pmatrix}$$

	Carbon	Heat	C_1	C_2		
1	L	L	1	0	1	0
2	L	M	-1	1	-1	1
3	L	H	0	-1	0	-1
4	H	L	1	0	-1	0
5	H	M	-1	1	1	-1
6	H	H	0	-1	0	1

(3) $Y=\beta_0+\beta_1 d_1+\beta_2 d_2+\beta_3 d_1 d_2+\varepsilon$

(a)

A	B	d_1	d_2	E(Y)
1	1	1	1	
1	2	1	-1	
2	1	-1	1	
2	2	-1	-1	

Main effect of A

$H_0: \beta_1=0$

$H_0: \beta_2=0$

$H_0: \beta_3=0$

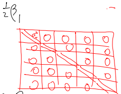
(B)

A	B	d_1	d_2	E(Y)
1	1	1	1	$\beta_0+\beta_1+\beta_2+\beta_3$
1	2	1	0	$\beta_0+\beta_1$
2	1	0	1	$\beta_0+\beta_2$
2	2	0	0	β_0

B

A	1	2	
1	$\beta_0+\beta_1+\beta_2+\beta_3$	$\beta_0+\beta_1$	$\beta_0+\beta_1+\frac{1}{2}(\beta_2+\beta_3)$
2	$\beta_0+\beta_2$	β_0	$\beta_0+\frac{1}{2}\beta_2$
	$\beta_0+\beta_2+\frac{1}{2}(\beta_1+\beta_3)$	$\beta_0+\frac{1}{2}\beta_1$	

(5) a) $Y_{ij}\sim N(\mu, \sigma_\tau^2+\sigma^2)$



b) Within School : random student selection
Outside School : random school selection $\Big\} \Rightarrow Y_{ij}$ indep.

$Cov(Y_{ij}, Y_{ij'}) = Cov(\cancel{\mu}+\tau_i+\varepsilon_{ij}, \cancel{\mu}+\tau_i+\varepsilon_{ij'})$

$= E(\tau_i^2) = \sigma_\tau^2$

c) $N\left(\mu, \frac{\sigma_\tau^2+\sigma^2}{K}\right)$

$Var(\bar{Y}_{i.}) = Var\left(\sum_{j=1}^{k}\frac{1}{k}(Y_{ij})\right)$

$=\frac{1}{k^2}Var\left(\sum_{j=1}^{k}(\mu+\tau_i+\varepsilon_{ij})\right)$

$=\frac{1}{k^2}Var\left(\sum_{j=1}^{k}\tau_i+\varepsilon_{ij}\right) = \frac{1}{k^2}Var\left(k\tau_i+\sum_{j=1}^{k}\varepsilon_{ij}\right)$

$=\frac{1}{k^2}(k^2\sigma_\tau^2+k\sigma^2) = \sigma_\tau^2+\frac{\sigma^2}{k}$

$\bar{Y}_{i.}=\frac{1}{k}\sum_{j=1}^{k}(\mu+\tau_i+\varepsilon_{ij})$

$=\mu+\tau_i+\bar{\varepsilon}_i$

(d)

$Cov(\bar{Y}_{i.}, Y_{ij}-\bar{Y}_{i.}) = Cov(\bar{Y}_{i.}, \cancel{\mu}+\cancel{\tau_i}+\varepsilon_{ij}-\cancel{\mu}-\cancel{\tau_i}-\bar{\varepsilon}_i)$

$= Cov(\cancel{\mu}+\tau_i+\bar{\varepsilon}_i, \varepsilon_{ij}-\bar{\varepsilon}_i)$ $=0$

$= E\{(\cancel{\tau_i}+\bar{\varepsilon}_i)(\varepsilon_{ij}-\bar{\varepsilon}_i)\} = Cov(\bar{\varepsilon}_i, \varepsilon_{ij}-\bar{\varepsilon}_i)$

(e) $\left(\frac{\sum_{i=1}^{g}(\bar{Y}_{i.}-\bar{Y}_{..})^2}{\sigma_\tau^2+\frac{\sigma^2}{k}}\right)\sim\chi^2(g-1)$

$E\left(\frac{SST}{\sigma^2+k\sigma_\tau^2}\right)=g-1$

so $E\left(\frac{SST}{g-1}\right)=\sigma^2+k\sigma_\tau^2$

$\frac{SST}{\sigma^2+k\sigma_\tau^2}=\frac{k\sum_{i=1}^{g}(\bar{Y}_{i.}-\bar{Y}_{..})^2}{\sigma^2+k\sigma_\tau^2}=\frac{k}{\sigma^2+k\sigma_\tau^2}\cdot\left(\sigma_\tau^2+\frac{\sigma^2}{k}\right)\cdot\frac{\sum(\bar{Y}_{i.}-\bar{Y}_{..})^2}{\sigma_\tau^2+\frac{\sigma^2}{k}}$

$\sim\chi^2(g-1)$

(f) $\frac{SSE}{\sigma^2}=\sum_{i=1}^{g}\boxed{\sum_{j=1}^{k}\frac{(\varepsilon_{ij}-\bar{\varepsilon}_i)^2}{\sigma^2}}$

Box $\sim\chi^2(k-1)$

$\sim\chi^2(g(k-1))$

(g) ESTIMATE $\frac{\sigma_\tau^2}{\sigma_\tau^2+\sigma^2}$

Expected MS

$E(MSE)=\sigma^2$

$E(MST)=\sigma^2+k\sigma_\tau^2$

if $\sigma_\tau^2=0$

(i) $F=\frac{\frac{SST}{\sigma^2+k\sigma_\tau^2}/(g-1)}{\frac{SSE}{\sigma^2}/g(k-1)}\sim F(g-1, g(k-1))$

$$\frac{\sum(\bar{Y}_{i.}-\bar{Y}_{..})^2\sim}{\sum_{i=1}^{g}\sum_{j=1}^{k}(Y_{ij}-\bar{Y}_{i.})^2\sim} \qquad \frac{g_1(\bar{Y}_1 \dots \bar{Y}_g)}{g_2(S_1, \dots S_g)}$$

$S_i=\sum_{j=1}^{k}(Y_{ij}-\bar{Y}_{i.})^2$

$\bar{Y}_i$ ind of S_i ?

$\bar{Y}_i$ ind $S_{i'}$, $i\neq i'$