STA 441: Data Analysis

Data Science

- Study design
- Data acquisition
- Data processing and perhaps pre-cleaning, yielding a data file.
- Data cleaning and description
- Data analysis and usually more cleaning.
- Interpretation, possibly with recommendations.
- Action

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Data File

- Rows are **cases**
- Columns are **variables**

1	2	2	0	78.0	65	80	39	English	Female	3	3	1
2	2	6	2	66.0	54	75	57	English	Female	3	3	1
3	2	4	4	80.2	77	70	62	English	Male	5	6	1
4	2	5	2	81.7	80	67	76	English	Female	2	2	1
5	2	4	4	86.8	87	80	86	English	Male	5	5	1
6	2	3	1	76.7	53	75	60	English	Male	3	3	1
7	2	3	2	85.8	86	81	54	Other	Female	2	2	1
8	2	4	3	73.0	75	77	17	English	Male	4	5	1
9	2	6	2	72.3	63	60	2	English	Male	4	4	1
10	2	8	6	90.3	87	88	76	English	Male	4	4	1
11	2	8	3	.	.	.	60	English	Male	1	2	1
12	2	6	4	.	.	.	61	Other	Female	1	1	1
13	.	.	.	87.2	84	83	54	English	Male	3	3	1
14	2	2	5	91.0	90	91	84	English	Male	5	5	1
15	2	3	1	72.8	53	74	.	English	Female	3	3	1
16	.	.	.	80.7	72	84	14	English	Male	3	3	1
17	2	5	0	82.5	82	85	75	Other	Female	2	2	1
18	2	4	6	91.5	95	81	94	English	Female	3	3	1
19	2	3	2	78.3	77	74	60	English	Female	3	3	1
20	.	.	.	74.5	0	85	.	English	Male	4	4	1
21	2	3	3	80.7	71	78	53	Other	Female	1	3	1
22	2	5	3	88.3	80	85	63	English	Female	3	3	1
23	2	4	2	76.8	82	64	82	Other	Female	2	2	1

Skipping ....

570	2	5	4	84.8	88	68	80	English	Male	1	1	1
571	2	4	3	78.3	83	84	56	English	Male	4	2	1
572	2	6	3	88.3	81	90	70	English	Female	5	5	1
573	2	3	1	.	.	.	.	English	Male	3	3	1
574	2	5	9	77.0	73	79	60	English	Female	2	2	1
575	.	.	.	78.7	80	73	.	English	Female	6	3	1
576	2	5	2	80.7	80	70	50	Other	Male	1	1	1
577	2	4	2	80.7	56	81	50	English	Female	2	2	1
578	2	4	3	.	.	.	78	Other	Female	4	4	1
579	1	6	1	82.2	80	86	61	English	Female	2	2	1

id	mcg	r	day	AML	AMS	AMld	PML	PMS	PMld	AMslp	PMslp	SWeight
1	198	1	1	0.6	.	.	0.8	.	.	.	.	
2	198	1	2	1.8	.	.	2.8	.	.	.	.	
3	198	1	3	4.7	1	.	6.1	1	.	.	.	
4	198	1	4	7.8	4	2.0	8.7	5	2.1	.	.	
5	198	1	5	11.2	6	1.8	12.1	7	2.0	.	.	
6	198	1	6	14.3	12	1.9	15.0	11	1.4	.	.	
7	198	1	7	17.5	12	2.1	18.5	13	1.6	.	.	
8	198	1	8	20.9	19	1.1	21.9	19	1.7	.	.	
9	198	1	9	24.0	22	1.6	25.2	22	1.3	.	.	
10	198	1	10	27.2	26	2.1	28.4	26	1.2	.	.	
11	198	1	11	30.7	28	1.4	32.3	28	1.5	.	.	
12	198	1	12	.	31	.	.	31	.	.	.	
13	198	1	13	.	37	.	.	36	.	.	.	
14	198	1	14	.	37	.	.	38	.	3.11	3.18	0.5996
15	198	2	1	0.5	.	.	0.6	.	.	.	.	
16	198	2	2	1.4	.	.	2.3	.	.	.	.	
17	198	2	3	4.15	1	.	5.6	1	.	.	.	
18	198	2	4	7.4	2	2.0	8.7	4	2.1	.	.	
19	198	2	5	10.8	5	2.2	12.0	8	2.0	.	.	
20	198	2	6	14.2	10	1.7	15.3	13	1.6	.	.	
21	198	2	7	17.1	13	2.2	18.1	16	1.7	.	.	
22	198	2	8	21.3	18	1.1	22.2	18	1.4	.	.	
23	198	2	9	24.4	27	1.4	25.6	24	1.2	.	.	
24	198	2	10	27.6	26	2.1	28.8	28	1.2	.	.	
25	198	2	11	31.2	29	1.9	32.5	29	1.3	.	.	
26	198	2	12	.	33	.	.	36	.	.	.	
27	198	2	13	.	38	.	.	41	.	.	.	
28	198	2	14	.	42	.	.	42	.	3.21	3.26	0.6040

Variables can be

- Quantitative - representing amount of something, like Income, BP, BMI, GPA (?)
- Categorical - Codes represent category membership, like Gender, Nationality, Marital status, Alive vs. dead

Variables can be

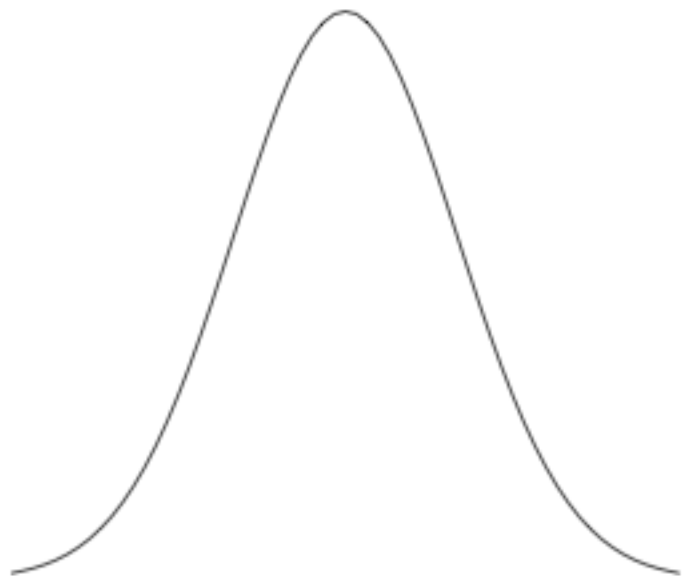
- Explanatory: Predictor or cause (contributing factor)
- Response: Predicted or effect

We will often pretend that our data represent a **random sample** from some **population**. We will carry out formal procedures for making inferences about this (usually fictitious) population, and then use them as a basis for drawing conclusions about the data.

- **Statistics**: Numbers that can be calculated from sample data

- **Parameters**: Numbers that could be calculated if we knew the whole population

Distribution = Population Histogram



Conditional Distribution

For each value x of the explanatory variable X, there is a separate distribution of the response Variable Y. This is called the conditional distribution of Y given $X=x$.

Example: Conditional distribution of height given Gender = F.

Definition of “Related”

- We will say that the explanatory and response variables are **unrelated** if the conditional distribution of the response variable is identical for each value of the explanatory variable.
- If the distribution of the response variable <u>does</u> depend on the value of the explanatory variable, we will describe the two variables as **related**.

Testing Statistical Significance

- Are explanatory variable and response variable "really" related?

- **Null Hypothesis**: They are unrelated in the population.

Reasoning

Suppose that the explanatory and response variables are actually unrelated in the population. If this null hypothesis is true, what is the probability of obtaining a sample relationship between the variables that is as strong or stronger than the one we have observed? If the probability is small (say, $p < 0.05$), then we describe the sample relationship as **statistically significant**, and it is socially acceptable to discuss the results.

P-value

- The probability of getting our results (or better) just by chance.

- The minimum significance level at which the null hypothesis can be rejected.

We can be wrong

- Type I error: H_0 is true, but we reject it
- Type II error: H_0 is false, but we fail to reject it

Power is the probability of *correctly* rejecting H_0

- Power = 1 - P(Type II Error)

- Power increases with true strength of relationship, and with sample size

- Power can be used to select sample size in advance of data collection

Confidence Interval: Pair of numbers chosen so that the probability they will enclose the parameter (or function of parameters) is large, like 0.95

Should we Accept H_0?

- When the results are not statistically significant, usually we will say that the data do not provide enough evidence to conclude that the variables are related.
- See text for more details.

Many statistical methods assume **Independent Observations**

- Simple random sampling
- Cases are not linked, do not "communicate"
- If the design involves non-independence, allow for it.

Elementary Tests

- Independent (two-sample) t-test
- Matched (paired) t-test
- One-way ANOVA
- Simple regression and correlation
- Chi-square test of independence

Independent t-test: Compare two means

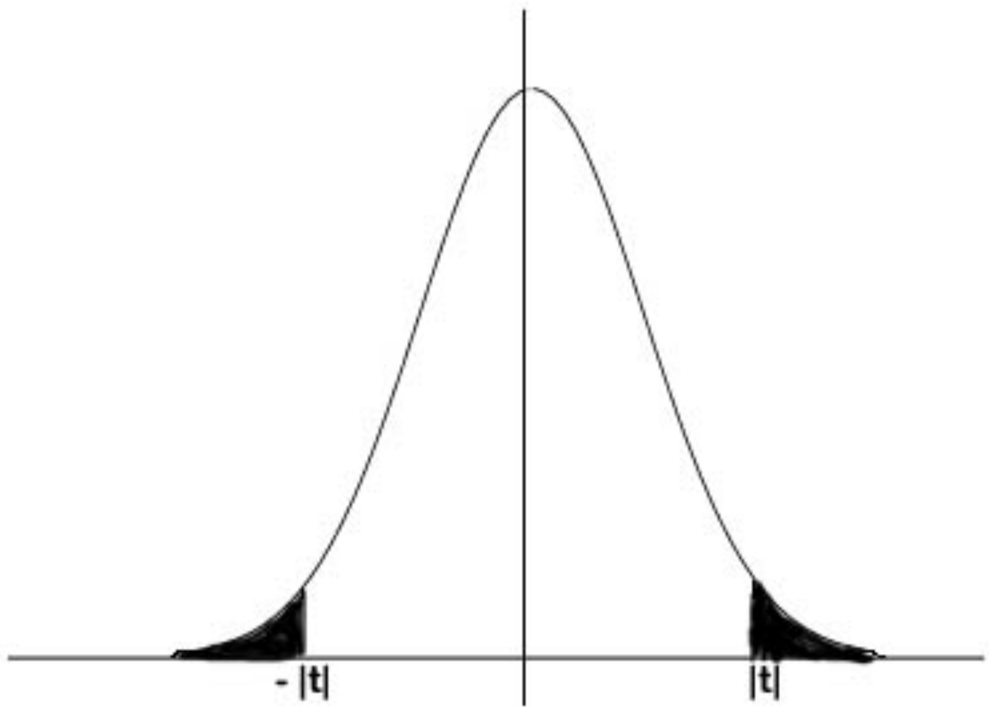
Data Plan	Productivity Rating
A	6.2
A	2.7
B	5.9
A	7.4
B	1.5
. . .	. . .

Model (Assumptions) for the independent t-test

- Random sampling, independently from two normal populations
- Possibly different population means
- Same population variance

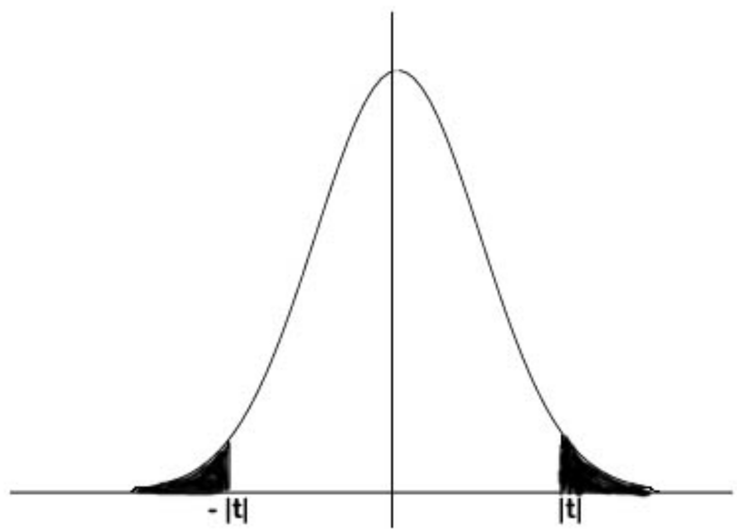
- Null hypothesis: Population means equal

Two-tailed tests and p-values only!



But we will always draw directional conclusions when possible

- Look at the sign of the regression coefficient
- Look at the sample means
- Look at the sample percentages



Robustness of the two-sample t-test

- Normality does not matter much if both samples are large
- Equal variance does not matter much if both samples are large and nearly equal in size
- Independent observations: Important

Matched (paired) t-test

Taste1	Taste2	Difference
10	8	2
7	7	0
3	4	-1
7	8	-1
6	5	1
…	…	…

Within versus between cases

- Between: A case contributes exactly one explanatory variable and one response variable value

- Within: A case contributes several pairs (explanatory variable, response variable) - usually one pair for each value of the explanatory variable

Model assumptions for matched t-test

- Random sampling of pairs
- Differences are normally distributed (satisfied if both measurements are normal)

Matched t-test

- Null Hypothesis: Mean difference equals zero
- Just a one-sample t-test applied to differences
- Can have more power than an inappropriate independent t-test

Robustness of matched t-test

- For large samples, normality does not matter
- Independent observations matter a lot

One-way analysis of variance

- Could call it “one-factor”
- Could call it “ANOVA”
- Extension of independent t-test: More than two values of the explanatory variable

- There are several within-cases versions - not elementary

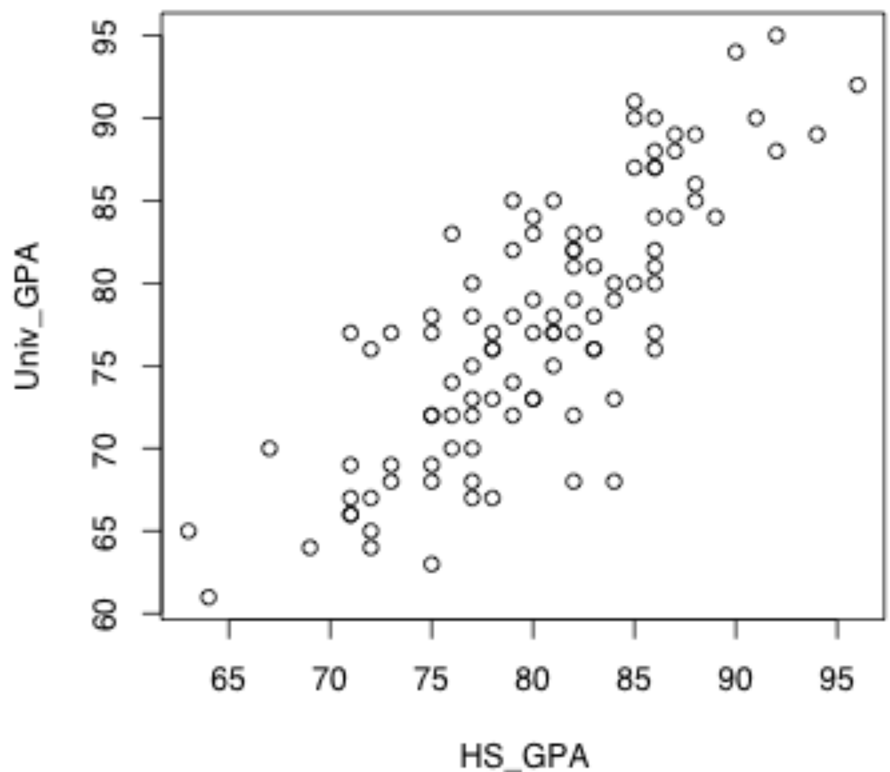
Simple regression and correlation

- Simple means one explanatory variable
- response variable quantitative
- explanatory variable usually quantitative too

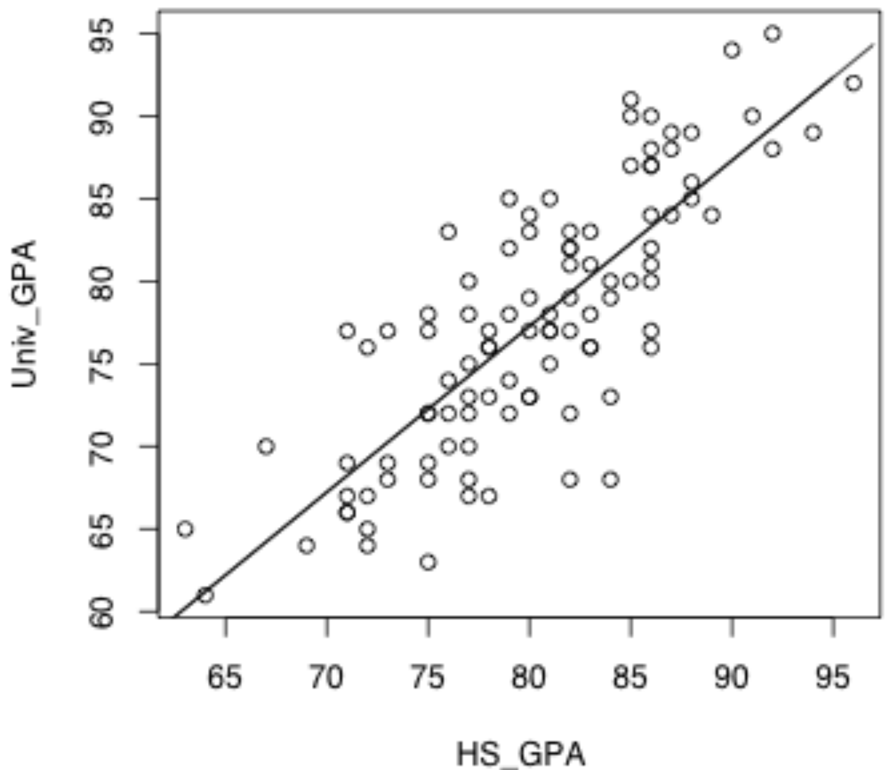
Simple regression and correlation

High School GPA	University GPA
88	86
78	73
87	89
86	81
77	67
…	…

Scatterplot



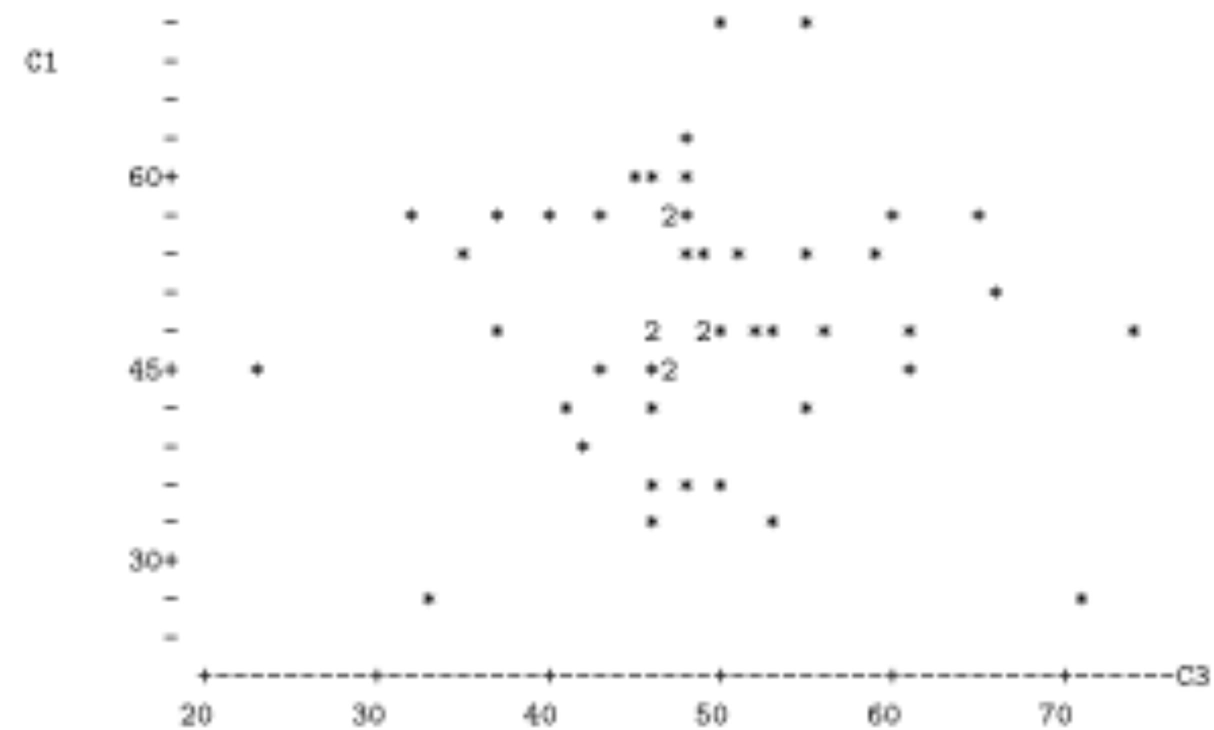
Least squares line



Correlation coefficient *r*

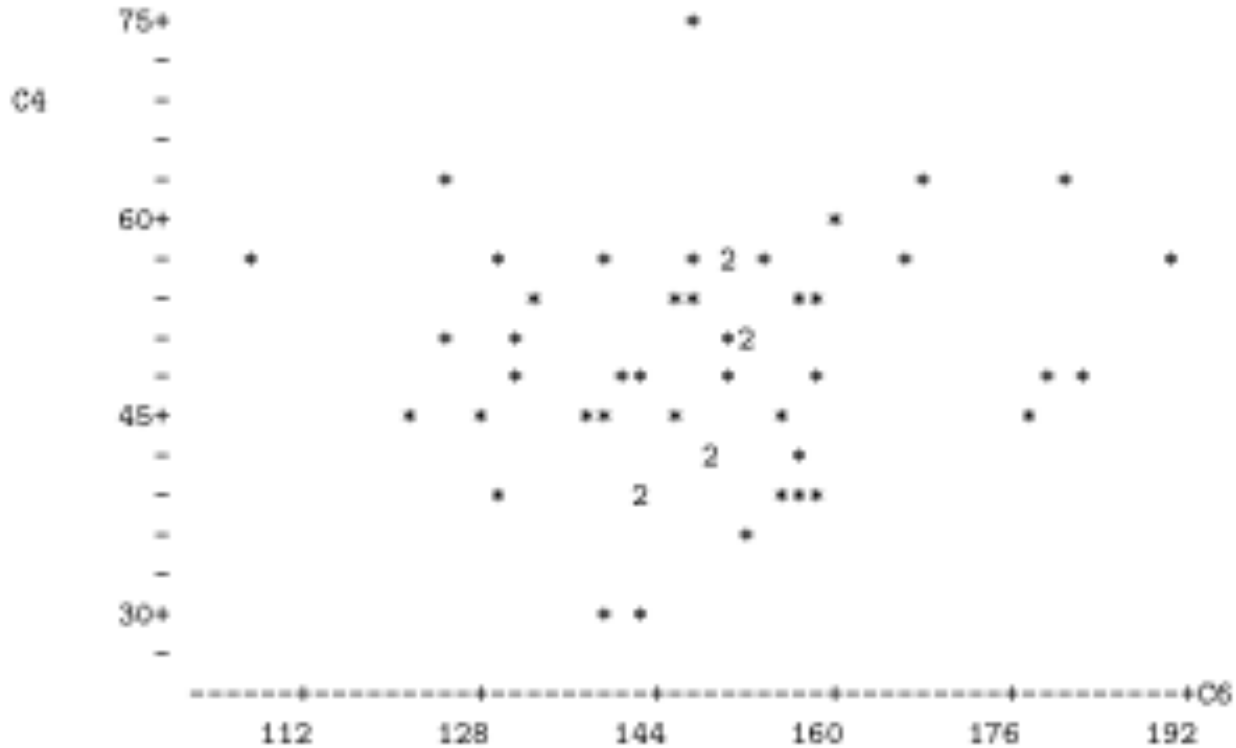
- $-1 \le r \le 1$
- $r = +1$ indicates a perfect positive linear relationship. All the points are exactly on a line with a positive slope.
- $r = -1$ indicates a perfect negative linear relationship. All the points are exactly on a line with a negative slope.
- $r = 0$ means no *linear* relationship (curve possible). Slope of least squares line = 0
- r^2 = proportion of variation explained

r = 0.004



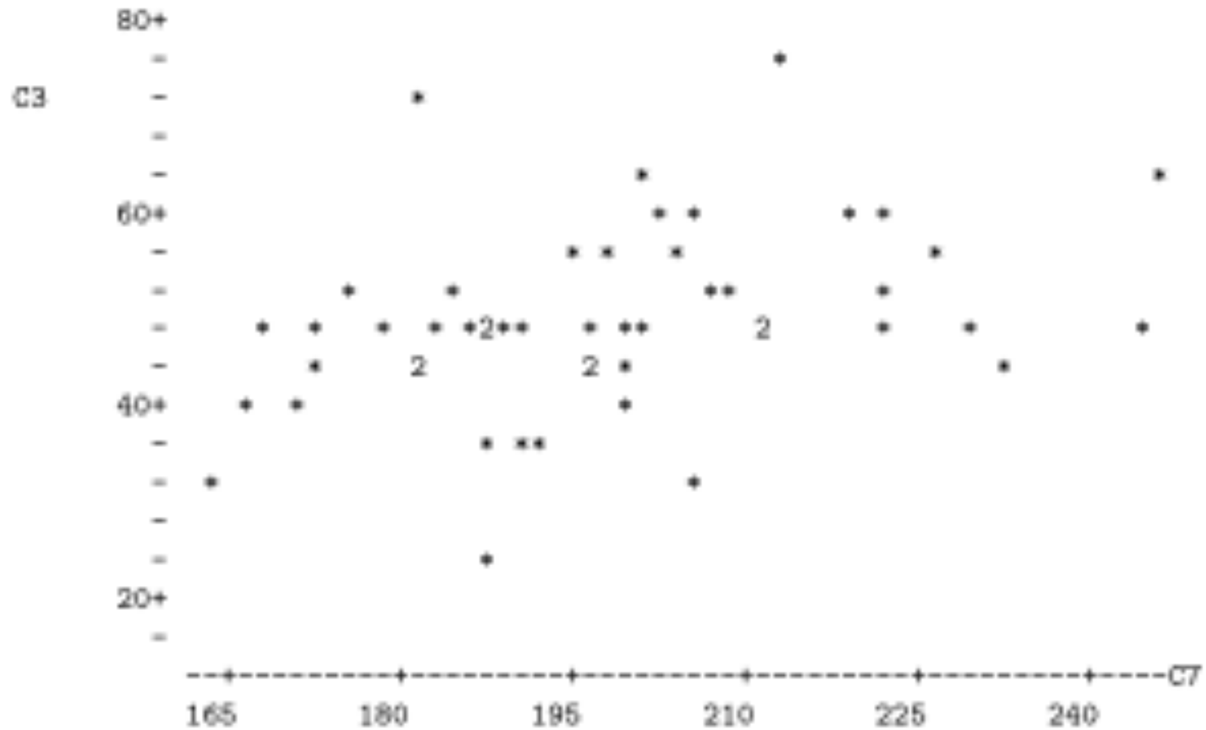
Correlation of C1 and C3 = 0.004

r = 0.112



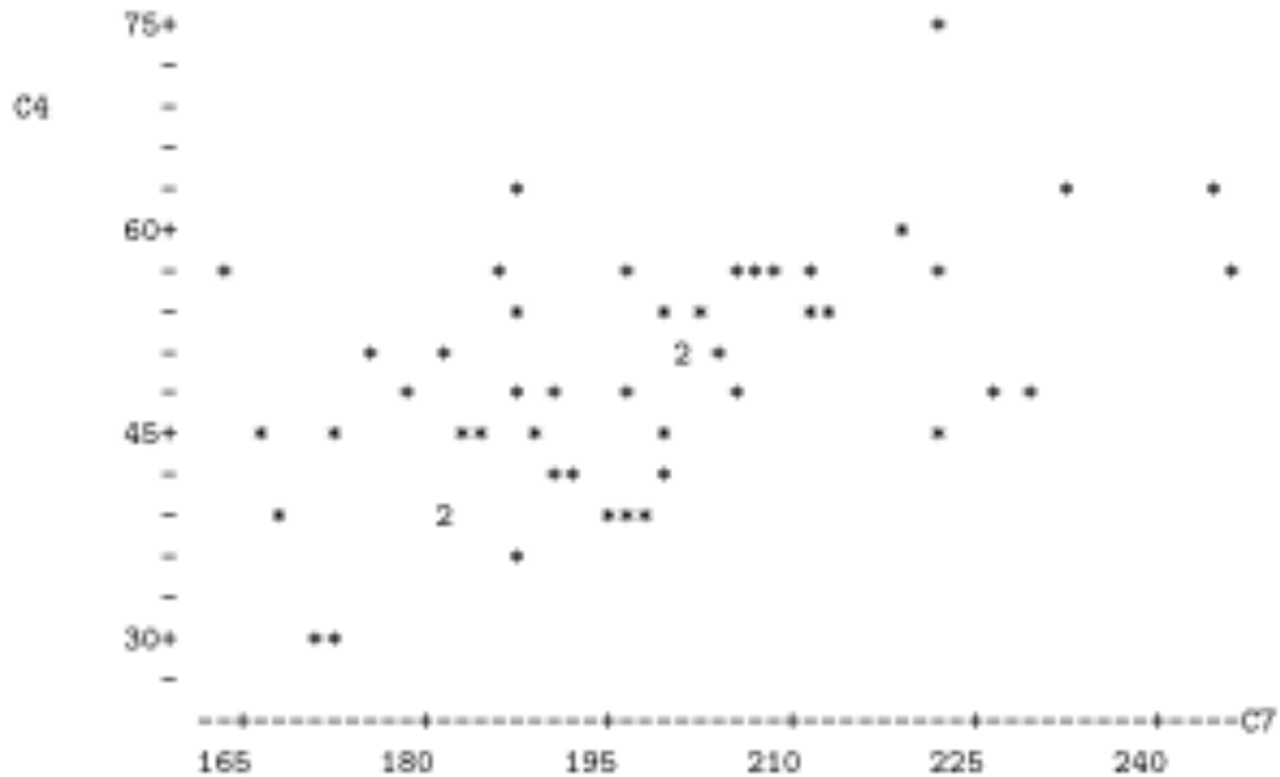
Correlation of C4 and C6 = 0.112

r = 0.368



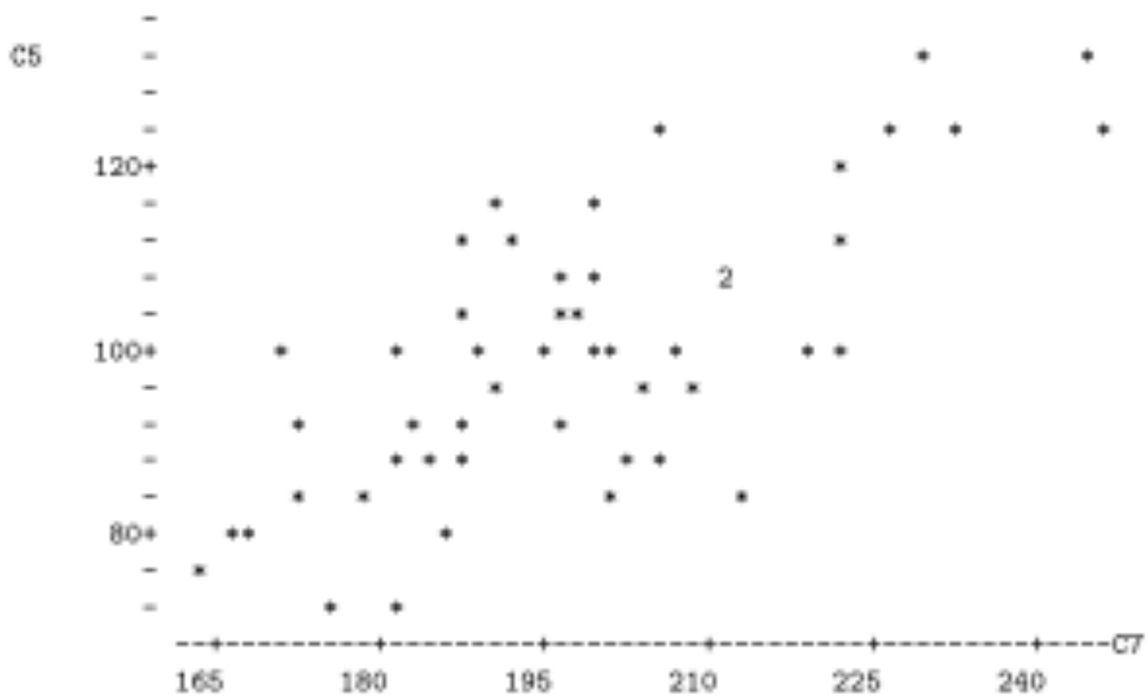
Correlation of C3 and C7 = 0.368

r = 0.547



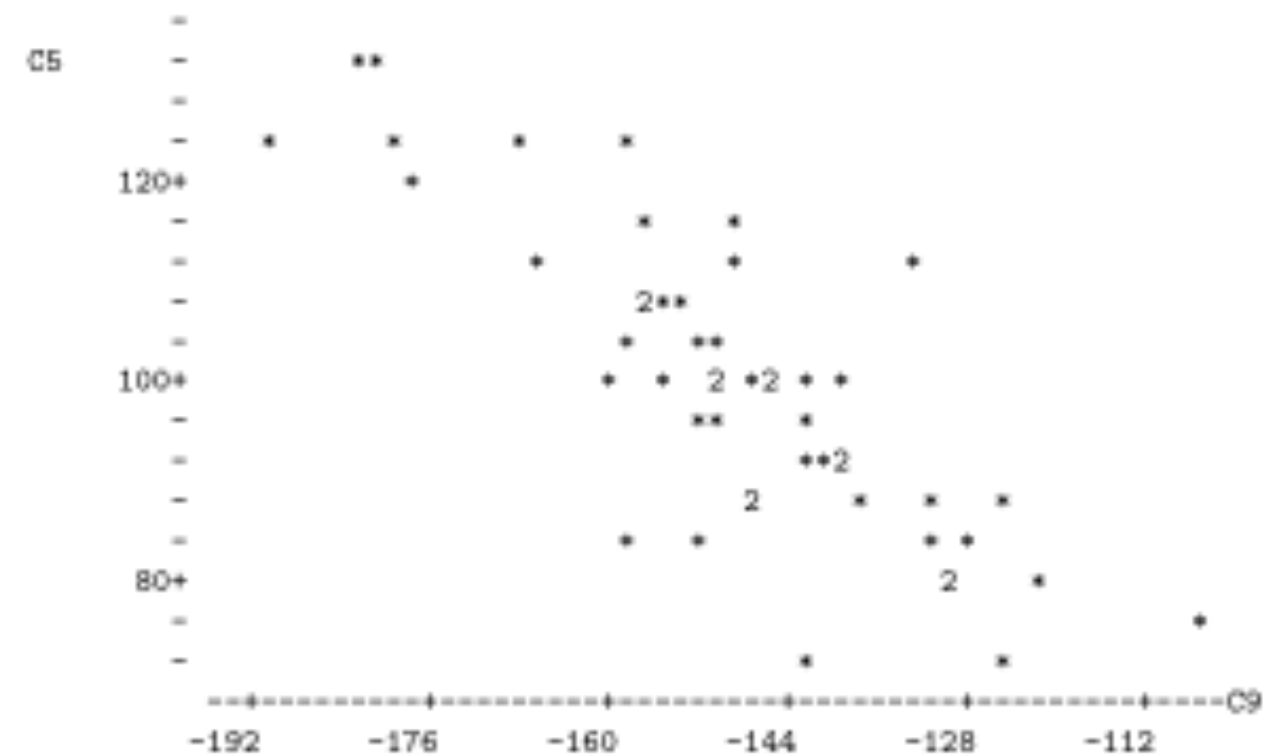
Correlation of C4 and C7 = 0.547

r = 0.733



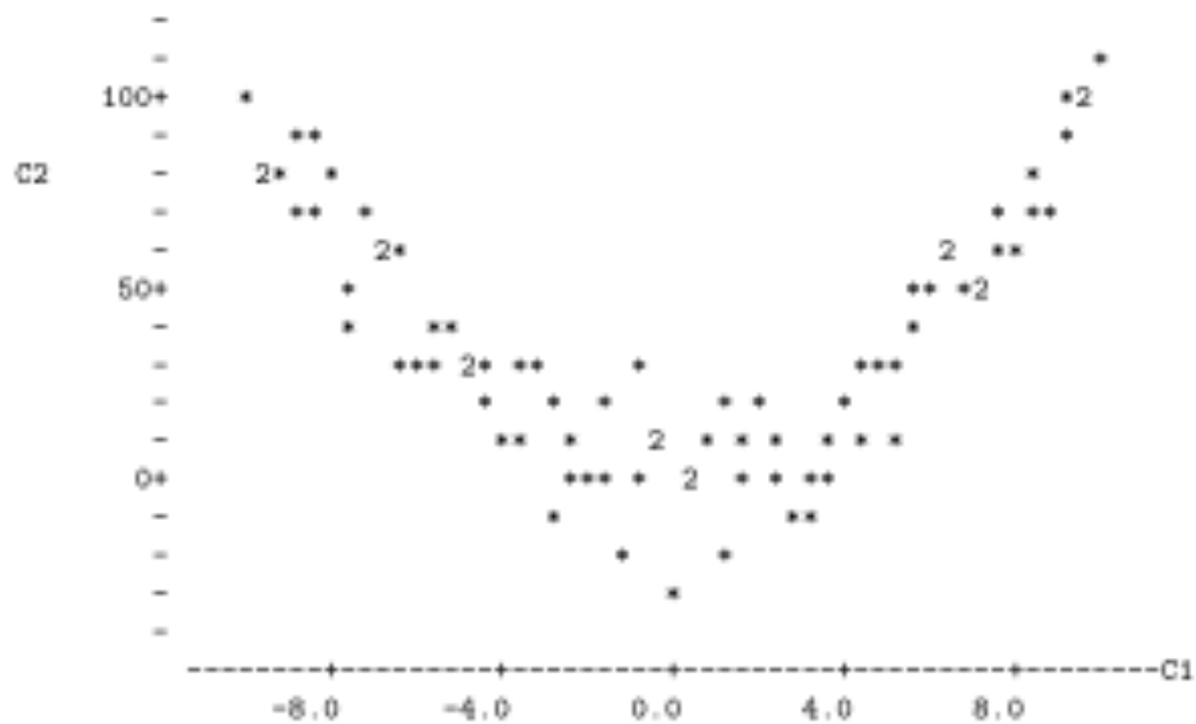
Correlation of C5 and C7 = 0.733

r = - 0.822



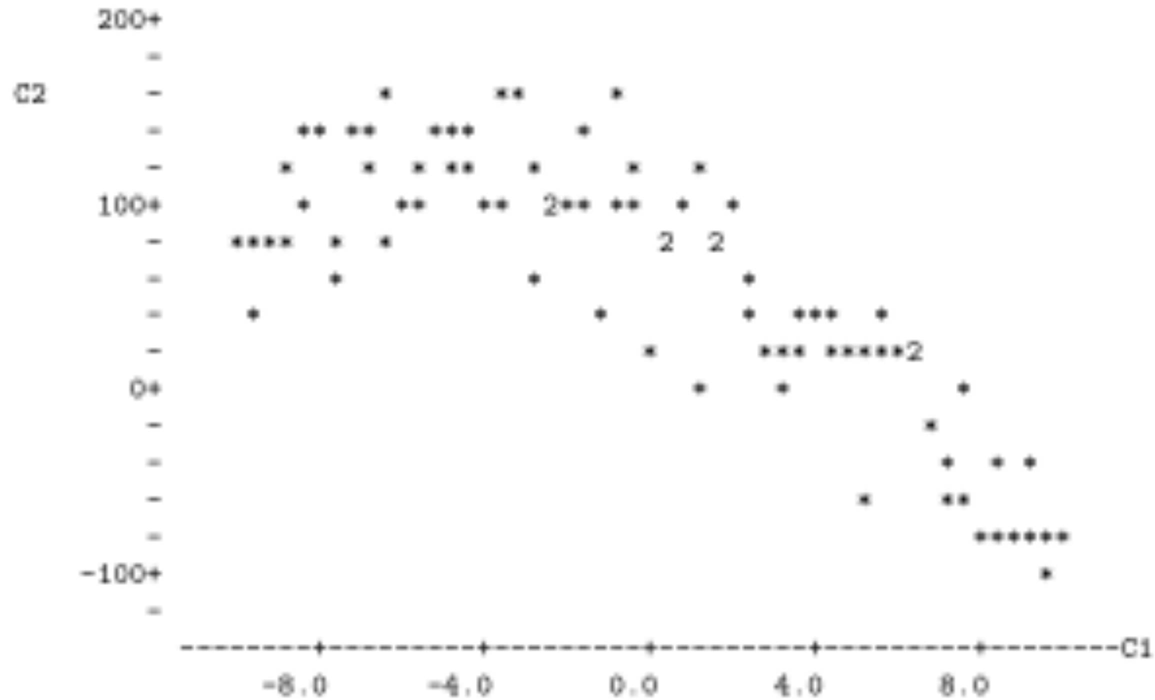
Correlation of C5 and C9 = -0.822

r = 0.025



Correlation of C1 and C2 = 0.025

r = - 0.811



Correlation of C1 and C2 = -0.811

Zero correlation = Horizontal least-squares line

$$\widehat{Y} = b_0 + b_1 X$$

$$b_1 = r\frac{s_y}{s_x} \text{ and } b_0 = \overline{Y} - b_1$$

Model assumptions for simple regression

- Random sampling of (X,Y) pairs
- Conditional distribution of response variable is normal for each explanatory variable value
- Maybe different mean, related to explanatory variable by equation of a straight line
- Variances all equal

Testing simple regression

- Null hypothesis: population slope = 0
- (This would make all the conditional distributions identical)
- Same as testing the significance of b_1
- Same as testing the significance of r

Robustness of simple regression test

- Normality does not matter much for large samples if the most influential observations are not too influential.
- Equal variance does not matter much if the number of observations at EACH value of X is large.
- Independent observations: Matters a lot

Chi-square test of independence: Both variables categorical

Music Type	**Stay on Hold?**
A	Yes
A	No
C	Yes
B	Yes
A	No
…	…

“Joint frequency distribution” or “contingency table” or “cross-tabulation” or “crosstab”

	Music Type			
	A	**B**	**C**	**D**
Yes	41	15	38	45
No	9	35	12	5

Model assumptions for the chi-squared test of independence

- The variable consisting of combinations of explanatory variable, response variable has a multinomial distribution
- “**Large**” random sample
- Rule of thumb: Lowest expected frequency no more than 5

- Independent observations: Important and often violated in practice.

Formula for the chi-square test

$$\chi^2 = \sum_{\text{cells}} \frac{(f_o - f_e)^2}{f_e}$$

- Even one very small expected frequency can make chisquare huge
- Smallest expected frequency no less than one (not 5) controls Type I error

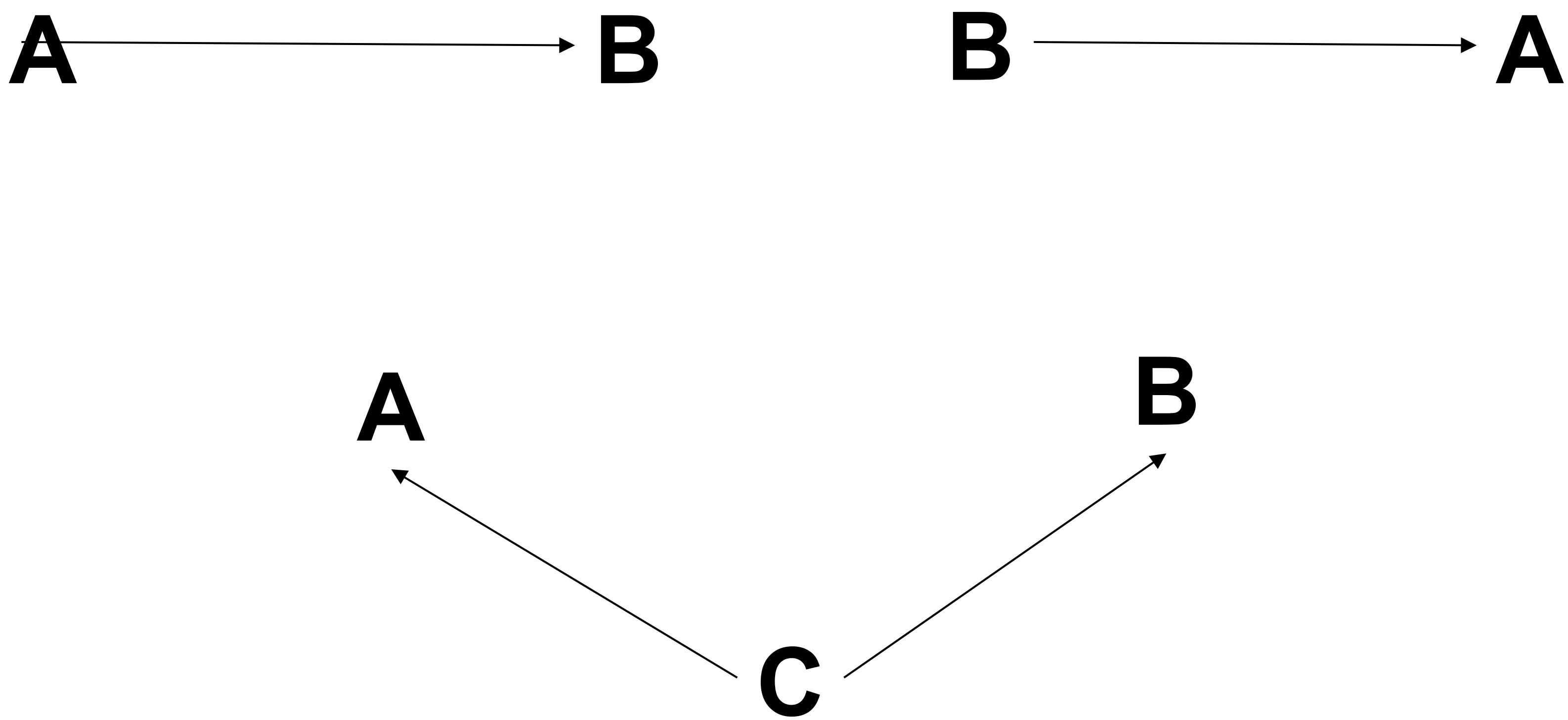
Why predict response variable from explanatory variable?

- There may be a practical reason for prediction (buy, make a claim, price of wheat).
- It may be “science.”

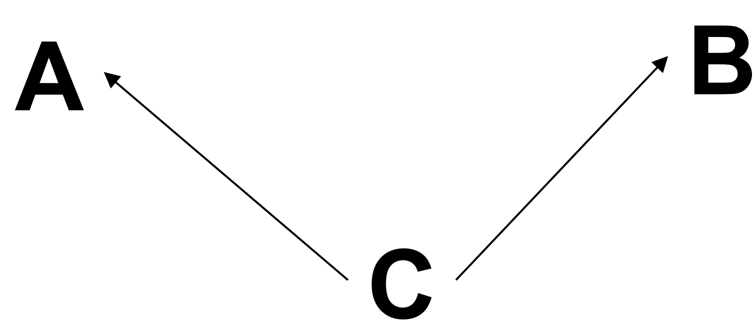
Young smokers who buy contraband cigarettes tend to smoke more.

- What is explanatory variable, response variable?

Correlation is not the same as causation



Confounding variable: A variable that contributes to both explanatory variable and response variable, causing a misleading relationship between them.



Mozart Effect

- Babies who listen to classical music tend to do better in school later on.
- Does this mean parents should play classical music for their babies?
- Please comment. (What is one possible confounding variable?)

Hypothetical study

- Subjects are babies in an orphanage awaiting adoption in Canada. All are assigned, but waiting for the paperwork to clear.
- They all wear headphones 5 hours a day. Randomly assigned to classical, rock, hip-hop or nature sounds. Same volume.
- Adoptive parents not informed.
- Assess academic progress in JK, SJ, Grade 4.
- Suppose there is a significant difference? What are some potential confounding variables?

Experimental vs. Observational studies

- **Observational**: explanatory variable, response variable just observed and recorded
- **Experimental**: Cases randomly assigned to values of explanatory variable
- Only a true experimental study can establish a causal connection between explanatory variable and response variable

- Maybe we should talk about observational vs experimental variables.
- Watch it: Confounding variables can creep back in.

Marking rule

- If you are interpreting the results of a purely observational study and you state an unqualified causal connection between explanatory and response variable, you lose a point.
- Examples:
  - Exercise affects arthritis pain.
  - Higher doses of Vitamin C lead to fewer colds.
  - Higher income produces greater average reported happiness.
  - More interaction with co-workers increases job satisfaction.
  - Textbook had a large effect.
  - Religion influences number of children.

Plain language is important

- If you can only be understood by mathematicians and statisticians, your knowledge is much less valuable.
- Often a question will say “Give the answer in plain, non-statistical language.”
- This means if x is income and y is credit card debt, you make a statement about income and average or predicted credit card debt, like “Customers with higher incomes tend to have less credit card debt.”
- If you use mathematical notation or words like null hypothesis, unbiased estimator, p-value or statistically significant, you will lose a lot of marks even if the statement is correct. Even avoid “positive relationship," and so on.

Plain language

- If the study is about fish, talk about fish.
- If the study is about blood pressure, talk about blood pressure.
- If the study is about breaking strength of yarn, talk about breaking strength of yarn.
- Assume you are talking to your boss, a former Commerce major who got a D+ in ECO220 and does not like to feel stupid.

We will be guided by tests with α = 0.05

- If we do not reject a null hypothesis like H_0: $\beta_1=0$, we will not draw a definite conclusion.
- Instead, say things like:
  - There is no evidence of a connection between blood sugar level and mood.
  - These results are not strong enough for us to conclude that attractiveness is related to mark in first-year Computer Science.
  - These results are consistent with no effect of dosage level on bone density.
- If the null hypothesis is not rejected, please do *not* claim that the drug has no effect, etc..
- In this we are taking Fisher's side in a historical fight between Fisher on one side and Neyman & Pearson on the other.
- Though we are guided by $\alpha = 0.05$, we *never* mention it when plain language is required.

No one-tailed tests

- In this class we will avoid one-tailed tests.
- Why? Ask what would happen if the results were strong and in the opposite direction to what was predicted.
- If the question asks for a null hypothesis and your answer has an inequality, it's wrong.
- But when H_0 is rejected, we still draw directional conclusions.

Directional conclusions

- Suppose x is income and y is credit card debt, and we test H_0: β_1=0 with a two-sided t-test.
- Say p = 0.0021 and b_1= 1.27.
- We say “Consumers with higher incomes tend to have more credit card debt.”
- Is this justified? We'd better hope so, or all we can say is “There is a connection between income and average credit card debt.”
- Then they ask: “What's the connection? Do people with lower income have more debt?”
- And you have to say “Sorry, I don't know.”
- It's a good way to get fired, or at least look silly.
- If a directional conclusion is possible and you only say “related,” you get half marks **at most**.

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