Within Cases ANOVA Part One[1]

STA441 Spring 2016

[1] See last slide for copyright information.

Within Cases

Example: A random sample of judges each tastes 4 wines and rates the flavour. Explanatory variable is type of wine.

- A case contributes a response variable value for more than one value of a categorical explanatory variable — usually all of them.
- It is natural to expect data from the same case to be correlated – *not* independent.
- For example, the same subject appears in several treatment conditions.
- Hearing study: How does pitch affect our ability to hear faint sounds? The same subjects will hear a variety of different pitch and volume levels (in a random order). They press a key when they think they hear something.

Student's Sleep Study (*Biometrika*, 1908)

First Published Example of a t-test

- Patients take two sleeping medicines several days apart.
- Half get A first, half get B first.
- Reported extra hours of sleep are recorded (difference from baseline).
- It's natural to subtract, and test whether the mean *difference* equals zero.
- That's what Gossett did.
- But some might do an independent t-test with $n_1 = n_2$.
- It's wrong, but is it harmful?

Matched pairs, testing $H_0 : \mu_1 = \mu_2$

Independent *v.s.* Matched t-test

- If population covariance between the two measurements is positive, Type I error probability of both tests is 0.05, but matched t-test has better power.
  - Each case serves as its own control.
  - Many unknown influences are removed by subtraction.
  - This makes the analysis more precise.
- If population covariance between measurements is *negative*, independent t-test has Type I error probability greater than 0.05.
  - Matched t-test still has the correct Type I error probability.
  - Negative covariance is unlikely to happen in most real situations.

Within-cases Terminology

You may hear terms like

- **Longitudinal**: The same variables are measured repeatedly over time. Usually there are lots of variables, including categorical ones, and large samples. If there's an experimental treatment, its usually once at the beginning, like a surgery. Longitudinal studies basically track what happens over time.
- **Repeated measures**: Usually, the same subjects experience two or more experimental treatments. Usually quantitative response variables, and often small samples.

Archery Example: Bow and Arrow

Two within-cases factors

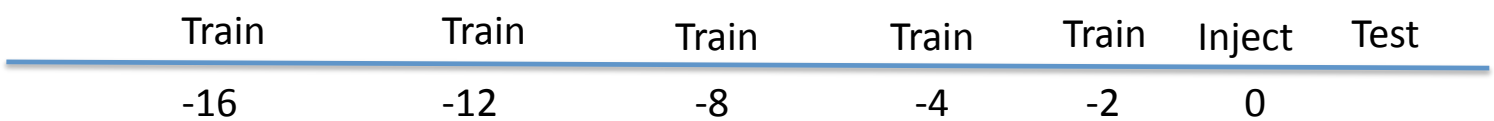
- Cases are archers. There are n archers.
- Test two bows, three arrow types.
- Warmup, then each archer takes 10 shots with each Bow-Arrow combination — 60 shots.
- In a different random order for each archer, of course.
- $Y_{i,1}, \ldots, Y_{i,6}$ are mean distances from arrow tip to centre of target, for $i = 1, \ldots, n$.
- Each $Y_{i,j}$ is based on 10 shots.
- $E(Y_{i,j}) = \mu_j$ for $j = 1, \ldots, 6$.

One Between, One Within

- Grapefruit study
- Within stores factor: Three price levels
- Between-stores factor: Incentive program for produce managers (Yes-No)

Monkey Study

- Train monkeys on discrimination tasks, at 16, 12, 8, 4 and 2 weeks prior to treatment. Different task each time, equally difficult (randomize order).
- Treatment is to block function of the hippocampus (with drug, not surgery), re-tested. Get 5 scores for each monkey.



- 11 randomly assigned to treatment, 7 to control
- Treatment is between, time elapsed since training is within.

Advantages of Within-cases Designs

If measurement of the response variable does not mess things up too much

- Convenience (sometimes)
- Each case serves as its own control. A huge number of extraneous variables are automatically held constant. The result can be a very sensitive analysis.
- For some models, you can have lots of measurements on just a few subjects — if you are willing to make some assumptions.

Three Main Approaches

For normal response variables

- Classical Mixed model
- Multivariate
- Covariance Structure

Classical Mixed Model Approach

- "Case" (or Subject) is one of the factors.
- Case is a *random effects* factor that is *nested* within combinations of the between-cases factors, and *crosses* the within-cases factors.
- Uses a mixed model ANOVA.
- F-tests depend on balanced experimental designs.

Multivariate Approach

- Multivariate methods allow the analysis of multiple response variables at the same time.
- The humble matched t-test has a multivariate version (Hotelling's t-squared).
- Simultaneously test whether the means of several *differences* equal zero.
- Like rating of Wine One minus Wine Two, Wine Two minus Wine Three, and Wine Three minus Wine Four.
- When there are also between-subjects factors (like nationality of judge), use multivariate regression methods.
- It's very attractive, but applies mostly to the normal case.
- The covariance structure approach is limited to the normal case too, but is more versatile.
- More on the covariance structure approach later.

Classical Mixed Model Approach

Repeating ...

- Case (or Subject) is one of the factors.
- Case is a *random effects* factor, because cases are assumed to be a randdom sample.
- Case is *nested* within combinations of the between-cases factors.
- Case *crosses* the within-cases factors.

No interactions of cases with other factors

A technical issue

- Cases (subjects) is a random effects factor.
- Models almost never include interactions between cases and other factors.
- This may not be realistic.
- Why assume it?
- Because with all possible interactions, $SSE = 0$ and $n - p = 0$ (details omitted).

Pictures of crossing and nesting

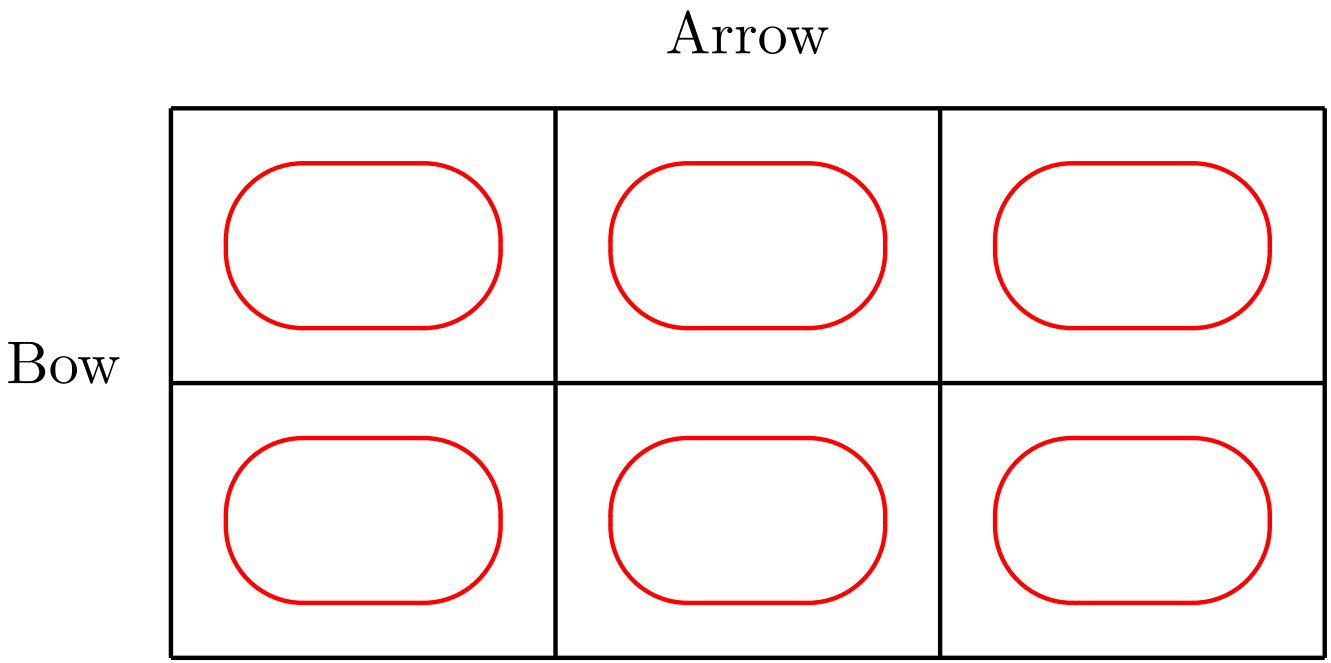
Cases (subjects) is a random effects factor nested within combinations of the between-cases factors and crossing the within-cases factors.

- Recall the archery example – two bow types, three arrow types.
- Suppose each archer only used one type of bow and one type of arrow.
- Make a diagram showing the nesting/crossing of cases.

Both Factors between

Make a diagram showing the nesting/crossing of cases.

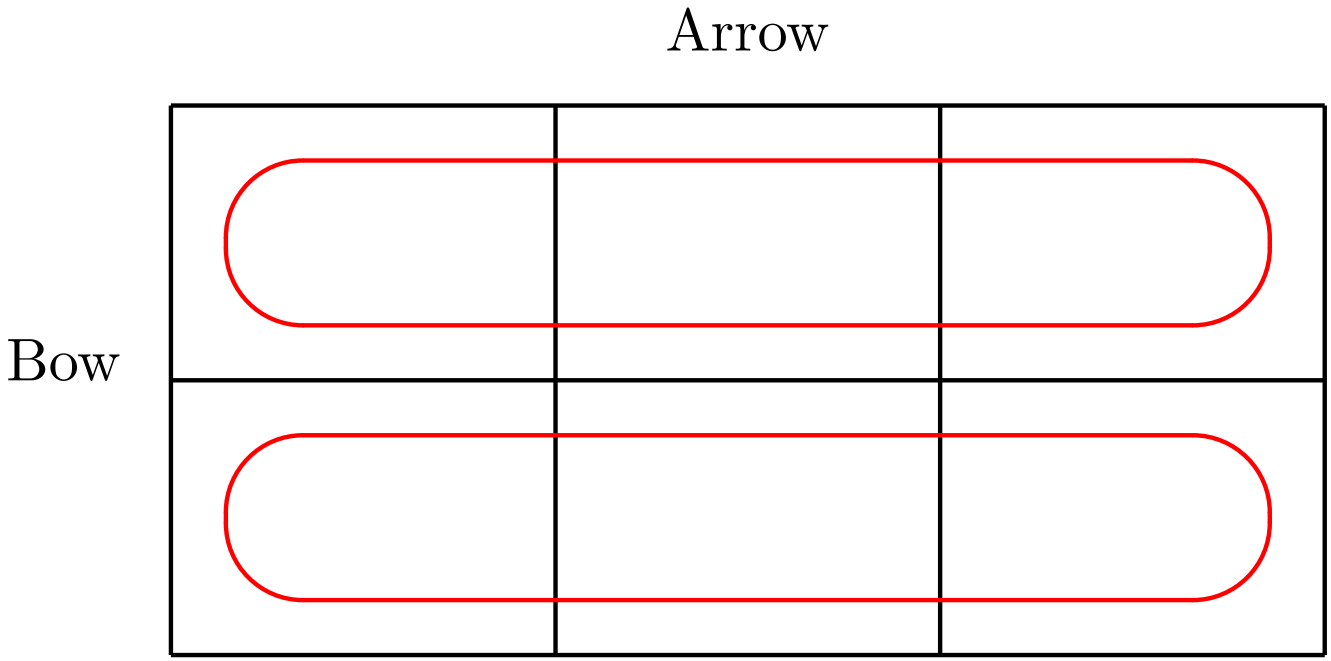
- Each archer only uses one type of bow and one type of arrow.
- Both factors are between cases.
- Cases are nested within both bow and arrow.



One factor between and one within

Make a diagram showing the nesting/crossing of cases.

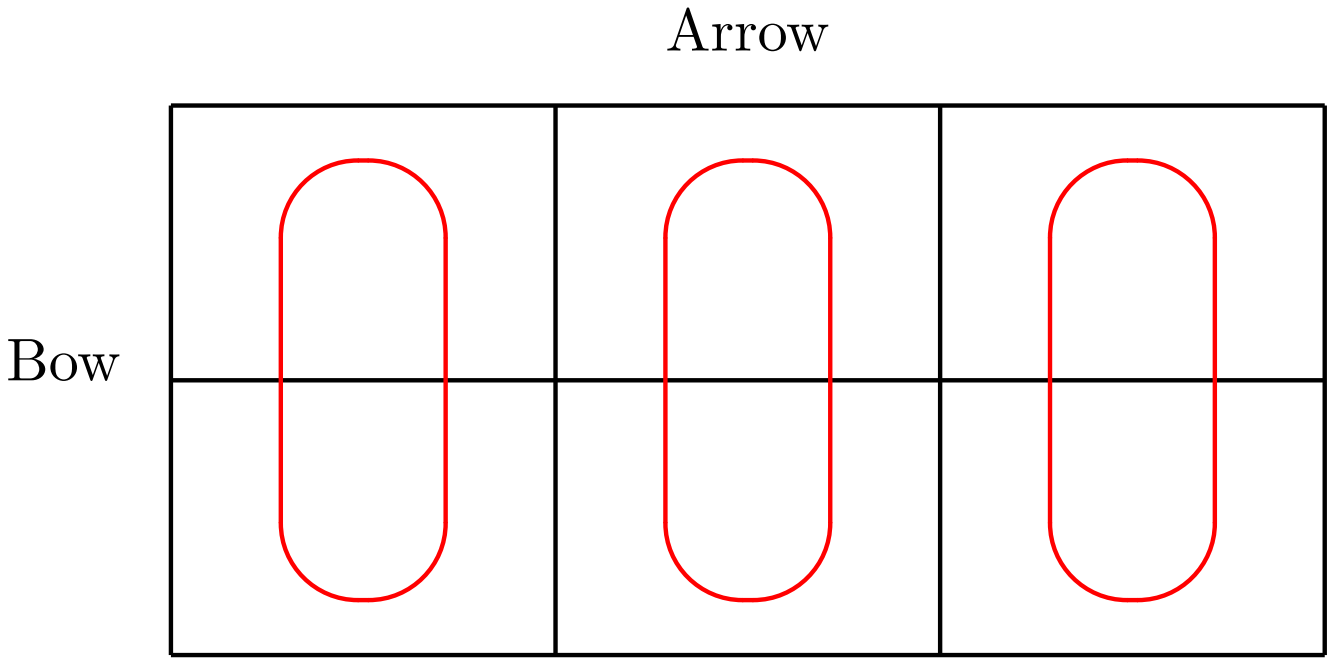
- Suppose each archer only uses one type of bow, but all 3 types of arrow.
- Bow is between cases, arrow is within (repeated measures on arrow).
- Cases are nested within bow, but cross arrow.



Another one factor between and one within

Make a diagram showing the nesting/crossing of cases.

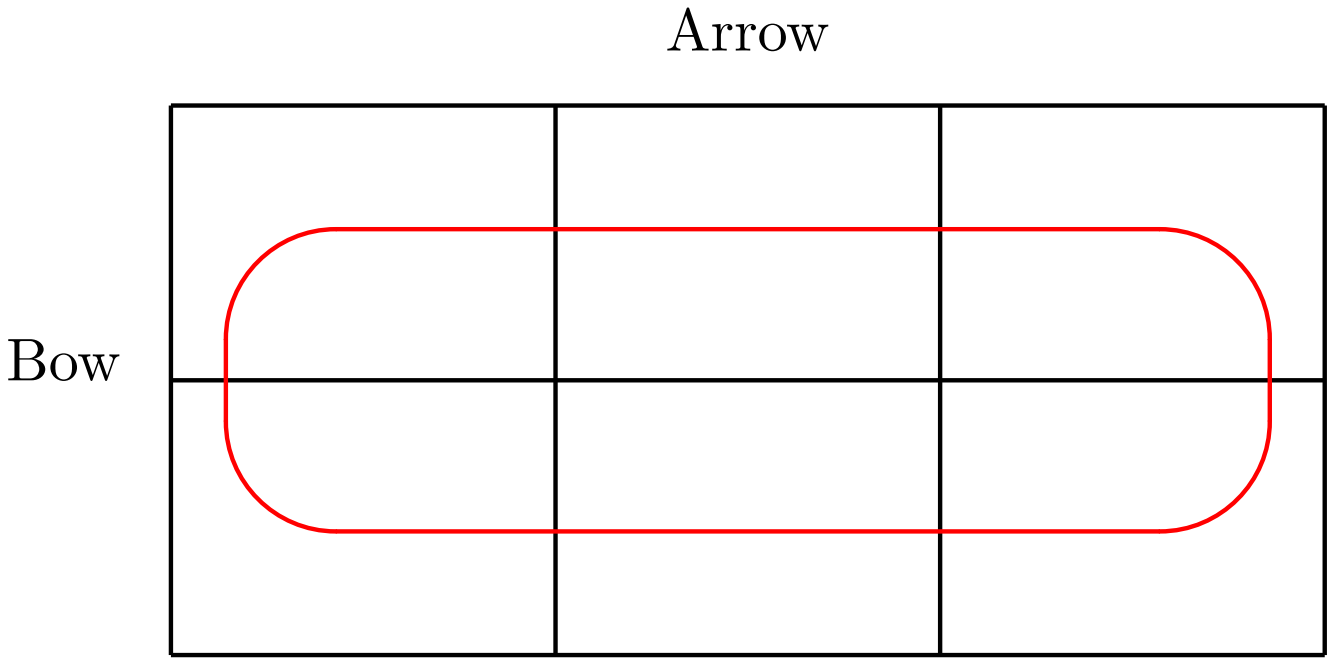
- Suppose each archer uses both types of bow, but only one type of arrow.
- Bow is within cases, Arrow is between (repeated measures on Bow).
- Cases are nested within Arrow, but cross Bow.



Both factors within

As in the original example

- Each archer uses both types of bow and all three types of arrow.
- Both factors are within cases (repeated measures on both Bow and Arrow).
- Cases cross both Bow and Arrow.



One More Example

Without a picture

- Experienced archers and beginners try both bows and all three arrow types.
- Experience is between cases, Bow and Arrow are within.
- Cases are nested within experience.

You draw the picture.

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