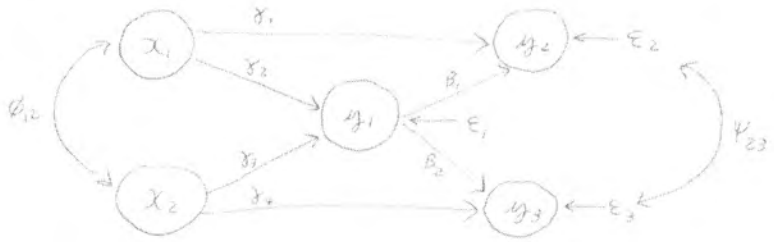
Name Jerry

Student Number _____

STA 431 Quiz 8

1. For the following latent variable path diagram,



(a) (3 points) Write the centered model equations in matrix form as $\mathbf{y} = \boldsymbol{\beta}\mathbf{y} + \boldsymbol{\Gamma}\mathbf{x} + \boldsymbol{\epsilon}$. The matrices should contain symbols indicated by the path diagram (and zeros).

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ \beta_1 & 0 & 0 \\ \beta_2 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \begin{pmatrix} \gamma_2 & \gamma_3 \\ \gamma_1 & 0 \\ 0 & \gamma_4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

(b) (2 points) Give the matrices $\boldsymbol{\Phi}_x = cov(\mathbf{x})$ and $\boldsymbol{\Psi} = cov(\boldsymbol{\epsilon})$. These matrices should contain symbols indicated by the path diagram (and zeros).

$$\underline{\Phi}_x = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{pmatrix} \qquad \Psi = \begin{pmatrix} \psi_{11} & 0 & 0 \\ 0 & \psi_{22} & \psi_{23} \\ 0 & \psi_{23} & \psi_{33} \end{pmatrix}$$

2. (5 points) For the R part of the assignment (last question), you simulated data from a structural equation model and estimated the parameters using `lavaan`. In the space below, write β_1 and $\widehat{\beta}_1$. These are numbers from your printout. On the printout, circle and label the numbers.

$\beta_1 = 0.75$, $\hat{\beta}_1 = 0.732$

Please attach your printout to the quiz paper. The printout should show your *complete* R input and output. Make sure your name and student number appear on the printout.

Assignment 8, Question 9

```
> # Simulate manipulation check design
> # Assignment 8, Question 9
>
> rm(list=ls())
> # source("rmvn.txt")
> source("https://www.utstat.toronto.edu/~brunner/openSEM/fun/rmvn.txt")
>
> # install.packages("lavaan", dependencies = TRUE) # Only need to do this once
> library(lavaan)

This is lavaan 0.6-11
lavaan is FREE software! Please report any bugs.
>
> # Set parameter values and sample size
> gamma = 1; beta1 = 0.75; beta2 = 1.25; phi = 2
> omega = 1; psi1 = 3; psi2 = 4; psi3 = 5; psi23 = 2
> n = 10000
>
> ################### Generate data ###################
> set.seed(888999)
> X = rnorm(n,0,sqrt(phi))
> epsilon1 = rnorm(n,0,sqrt(psi1))
> e = rnorm(n,0,sqrt(omega))
> V23 = rbind(c(psi2,psi23),
+             c(psi23,psi3))
> er = rmvn(n,c(0,0),V23)
> epsilon2 = er[,1]
> epsilon3 = er[,2]
>
> Y1 = gamma*X + epsilon1
> Y2 = beta1*Y1 + epsilon2
> Y3 = beta2*Y1 + epsilon3
> V = Y1 + e
> Mcheck = cbind(X,V,Y2,Y3)
> #####################################################
>
> cor(Mcheck)
           X         V        Y2        Y3
X  1.0000000 0.5726864 0.3931787 0.4845121
V  0.5726864 1.0000000 0.5790716 0.7084621
Y2 0.3931787 0.5790716 1.0000000 0.7146340
Y3 0.4845121 0.7084621 0.7146340 1.0000000
```

```
> ####### Fit model with lavaan
>
> mod1 = 'Y1 ~ gamma*X
+         Y1 =~ 1.0*V + beta1*Y2 + beta2*Y3
+         # Variances and covariances
+         X ~~ phi*X     # Var(X) = phi
+         V ~~ omega*V   # Var(e) = omega
+         Y1 ~~ psi1*Y1  # Var(epsilon1) = psi1
+         Y2 ~~ psi2*Y2  # Var(epsilon2) = psi2
+         Y3 ~~ psi3*Y3  # Var(epsilon3) = psi3
+         Y2 ~~ psi23*Y3 # Cov(epsilon2,epsilon3) = psi23
+        ' # End of mod1
>
> fit1 = lavaan(mod1, data=Mcheck)
> # summary(fit1)
> parameterEstimates(fit1)
   lhs op rhs label   est    se       z pvalue ci.lower ci.upper
1   Y1  ~   X gamma 0.983 0.014 69.859      0    0.955    1.010
2   Y1 =~   V       1.000 0.000     NA     NA    1.000    1.000
3   Y1 =~  Y2 beta1 0.732 0.013 56.490      0    0.707    0.758
4   Y1 =~  Y3 beta2 1.247 0.018 68.087      0    1.211    1.283
5    X ~~   X   phi 2.010 0.028 70.711      0    1.954    2.066
6    V ~~   V omega 0.956 0.060 15.817      0    0.838    1.075
7   Y1 ~~  Y1  psi1 3.020 0.073 41.500      0    2.877    3.162
8   Y2 ~~  Y2  psi2 3.997 0.068 58.848      0    3.864    4.130
9   Y3 ~~  Y3  psi3 5.164 0.118 43.867      0    4.933    5.395
10  Y2 ~~  Y3 psi23 2.088 0.075 27.934      0    1.941    2.234

>
> # Try mod2 with just one measurement of Y1
> # and regressions connecting Y1 to Y2 and Y3
>
> mod2 = 'Y1 ~ gamma*X
+         Y2 ~ beta1*Y1
+         Y3 ~ beta2*Y1
+         Y1 =~ 1.0*V
+         # Variances and covariances
+         X ~~ phi*X     # Var(X) = phi
+         V ~~ omega*V   # Var(e) = omega
+         Y1 ~~ psi1*Y1  # Var(epsilon1) = psi1
+         Y2 ~~ psi2*Y2  # Var(epsilon2) = psi2
+         Y3 ~~ psi3*Y3  # Var(epsilon3) = psi3
+         Y2 ~~ psi23*Y3 # Cov(epsilon2,epsilon3) = psi23
+        ' # End of mod2
>
> fit2 = lavaan(mod2, data=Mcheck)
> parameterEstimates(fit2)
   lhs op rhs label   est    se       z pvalue ci.lower ci.upper
1   Y1  ~   X gamma 0.983 0.014 69.859      0    0.955    1.010
2   Y2  ~  Y1 beta1 0.732 0.013 56.490      0    0.707    0.758
3   Y3  ~  Y1 beta2 1.247 0.018 68.087      0    1.211    1.283
4   Y1 =~   V       1.000 0.000     NA     NA    1.000    1.000
5    X ~~   X   phi 2.010 0.028 70.711      0    1.954    2.066
6    V ~~   V omega 0.956 0.060 15.817      0    0.838    1.075
7   Y1 ~~  Y1  psi1 3.020 0.073 41.500      0    2.877    3.162
8   Y2 ~~  Y2  psi2 3.997 0.068 58.848      0    3.864    4.130
9   Y3 ~~  Y3  psi3 5.164 0.118 43.867      0    4.933    5.395
10  Y2 ~~  Y3 psi23 2.088 0.075 27.934      0    1.941    2.234
```