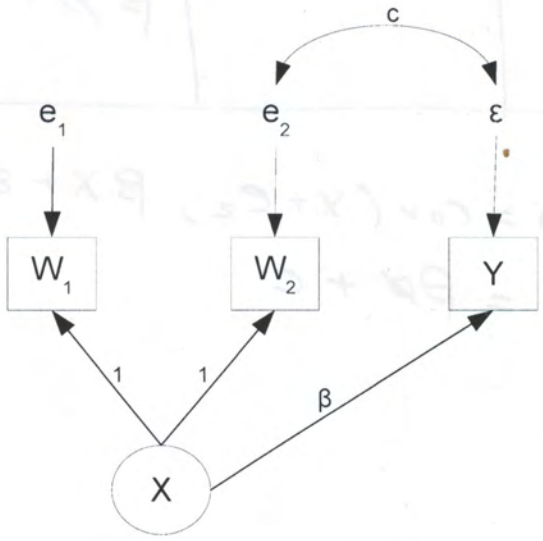
Name Jerry

Student Number _____

STA 431 Quiz 7

1. In this double measurement path diagram, data for W_2 and Y were collected in the same setting and at the same time. They were influenced by common omitted variables, and so their error terms are correlated.



(a) (1 point) Write the model equations. Just write three equations and nothing else.

$W_1 = X + e_1$ (Subscripts i are optional)

$W_2 = X + e_2$

$Y = \beta X + \varepsilon$

(b) (1 point) Using the notation $Var(X) = \phi$, $Var(e_1) = \omega_1$, $Var(e_2) = \omega_2$ and $Var(\epsilon) = \psi$, what is the parameter vector $\boldsymbol{\theta}$ for this model?

$\theta = (\phi, \omega_1, \omega_2, \beta, c, \varepsilon)$ (Of course order does not matter)

(c) (1 point) Does this model pass the test of the Parameter Count Rule? Answer Yes or No and give the numbers.

Yes: 6 equations in 6 unknowns

$W_1 = X + e_1$
$W_2 = X + e_2$
$Y = \beta X + \xi$

(d) (3 points) Give the covariance matrix of the observable data as a function of the model parameters. Show any necessary calculations.

$\Sigma =$

	W_1	W_2	Y
W_1	$\phi + w_1$	ϕ	$\beta\phi$
W_2	$\phi + w_2$		$\beta\phi + c$
Y			$\beta^2\phi + \psi$

$Cov(W_2, Y) = Cov(X + e_2, \beta X + \varepsilon)$
$= \beta\phi + c$

(No + even this work is actually required.)

(e) (2 points) Is the parameter β identifiable? If Yes, show it. If No, give two parameter vectors that have *different* values of β, but produce the same covariance matrix. A simple numerical example would be best.

Yes, identifiable: $\beta = \sigma_{13}/\sigma_{12}$

2. (2 points) For the R part of the assignment, you analyzed the Pig Birth data. In a path diagram of the model (which you do *not* have to give here), there would be a curved double-headed arrow connecting the measurement error terms for Questionnaire One.

   What is the value of the test statistic for testing whether the arrow is present? The answer is a number on your printout. Write the number in the space below. On your printout, circle the number and write "Question 2" beside it. *Do not answer this question if you do not have a printout for Question 3 of this weeks's assignment.*

$z = 5.389$

Please attach your printout to the quiz paper. The printout should show your *complete* R input and output. Make sure your name and student number appear on the printout.

Assignment 7, Question 3

```
> # a.
> pigs = read.table("http://www.utstat.toronto.edu/brunner/openSEM/data/openpigs2.data.txt")
> head(pigs); dim(pigs)
  nbreedersQ1 ngivebirthQ1 nbreedersQ2 ngivebirthQ2
1          69           45          89           67
2          52           24          85           41
3          48           33          68           34
4           7            4          30           30
5          41           37          54           35
6          35           23          57           48
[1] 114   4
>
> # b.
> S = var(pigs); S
             nbreedersQ1 ngivebirthQ1 nbreedersQ2 ngivebirthQ2
nbreedersQ1     691.3607     581.6943    348.5299     272.6710
ngivebirthQ1    581.6943     562.6853    260.0286     239.2279
nbreedersQ2     348.5299     260.0286    770.9685     370.8930
ngivebirthQ2    272.6710     239.2279    370.8930     334.1683

> # c. and d. are handwritten.
>
> # e.
>
> # install.packages("lavaan", dependencies = TRUE) # Only need to do this once
> library(lavaan)
This is lavaan 0.6-11
lavaan is FREE software! Please report any bugs.
>
> pigmod =
+     '#######################################################
+      # Latent variable model
+      # ---------------------
+      Lbirth ~ beta*Lbreeders
+      #
+      # Measurement model
+      # -----------------
+      Lbreeders =~ 1*nbreedersQ1  + 1*nbreedersQ2
+      Lbirth    =~ 1*ngivebirthQ1 + 1*ngivebirthQ2
+      #
+      # Variances
+      # ---------
+      Lbreeders ~~ phi*Lbreeders          # Var(Lbreeders) = phi
+      Lbirth ~~ psi*Lbirth                # Var(epsilon) = psi
+      nbreedersQ1  ~~ omega1*nbreedersQ1  # Var(e1) = omega1
+      ngivebirthQ1 ~~ omega2*ngivebirthQ1 # Var(e2) = omega2
+      nbreedersQ2  ~~ omega3*nbreedersQ2  # Var(e3) = omega3
+      ngivebirthQ2 ~~ omega4*ngivebirthQ2 # Var(e4) = omega4
+      #
+      # Covariances (between error terms)
+      # ---------------------------------
+      nbreedersQ1  ~~ omega12*ngivebirthQ1  # Cov(e1,e2) = omega12
+      nbreedersQ2  ~~ omega34*ngivebirthQ2  # Cov(e3,e4) = omega34
+      # Reliabilities of number of breeding sows for the two questionnaires
+      # -------------------------------------------------------------------
+      rel1 := omega1/(phi+omega1)
+      rel2 := omega3/(phi+omega3)
+     ' ###################### End of pigmod ########################
>
> fit1 = lavaan(pigmod, data=pigs)
> summary(fit1)
```

```
lavaan 0.6-11 ended normally after 120 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         9

  Number of observations                           114

Model Test User Model:

  Test statistic                                 0.087
  Degrees of freedom                                 1
  P-value (Chi-square)                           0.768

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  Lbreeders =~
    nbreedersQ1       1.000
    nbreedersQ2       1.000
  Lbirth =~
    ngivebirthQ1      1.000
    ngivebirthQ2      1.000

Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Lbirth ~
    Lbredrs (beta)    0.757    0.054   14.047    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .nbreedersQ1 ~~
   .ngvbrQ1 (om12)  308.539   57.249    5.389    0.000
 .nbreedersQ2 ~~
   .ngvbrQ2 (om34)  101.406   45.513    2.228    0.026

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    Lbredrs  (phi)  357.145   64.936    5.500    0.000
   .Lbirth   (psi)   33.634   10.861    3.097    0.002
   .nbrdrQ1 (omg1)  330.683   67.114    4.927    0.000
   .ngvbrQ1 (omg2)  321.255   54.160    5.932    0.000
   .nbrdrQ2 (omg3)  416.335   80.763    5.155    0.000
   .ngvbrQ2 (omg4)   93.000   35.566    2.615    0.009

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    rel1              0.481    0.070    6.877    0.000
    rel2              0.538    0.070    7.706    0.000

>
> # h.
>
> # MOM of beta: Really need to calculate Sigma to see this.
> 0.5*(S[1,4]+S[2,3])/S[1,3]
[1] 0.7642093
```

Question 2

```
>
> # i.
> parameterEstimates(fit1)
            lhs op                 rhs    label     est     se      z pvalue ci.lower ci.upper
1        Lbirth  ~           Lbreeders     beta   0.757  0.054 14.047  0.000    0.651    0.862
2     Lbreeders =~         nbreedersQ1            1.000  0.000     NA     NA    1.000    1.000
3     Lbreeders =~         nbreedersQ2            1.000  0.000     NA     NA    1.000    1.000
4        Lbirth =~        ngivebirthQ1            1.000  0.000     NA     NA    1.000    1.000
5        Lbirth =~        ngivebirthQ2            1.000  0.000     NA     NA    1.000    1.000
6     Lbreeders ~~           Lbreeders      phi 357.145 64.936  5.500  0.000  229.873  484.417
7        Lbirth ~~              Lbirth      psi  33.634 10.861  3.097  0.002   12.348   54.920
8   nbreedersQ1 ~~         nbreedersQ1   omega1 330.683 67.114  4.927  0.000  199.143  462.224
9  ngivebirthQ1 ~~        ngivebirthQ1   omega2 321.255 54.160  5.932  0.000  215.103  427.407
10  nbreedersQ2 ~~         nbreedersQ2   omega3 416.335 80.763  5.155  0.000  258.042  574.628
11 ngivebirthQ2 ~~        ngivebirthQ2   omega4  93.000 35.566  2.615  0.009   23.291  162.709
12  nbreedersQ1 ~~        ngivebirthQ1  omega12 308.539 57.249  5.389  0.000  196.334  420.744
13  nbreedersQ2 ~~        ngivebirthQ2  omega34 101.406 45.513  2.228  0.026   12.201  190.610
14         rel1 := omega1/(phi+omega1)     rel1   0.481  0.070  6.877  0.000    0.344    0.618
15         rel2 := omega3/(phi+omega3)     rel2   0.538  0.070  7.706  0.000    0.401    0.675

> # j. k. and L.  See above.
>
> # m. Compare precision of measurement.
> # H0: omega1=omega3 and omega2=omega4
> # For Wald tests: Wtest = function(L,Tn,Vn,h=0) # H0: L theta = h
> source("http://www.utstat.utoronto.ca/~brunner/Rfunctions/Wtest.txt")
>
> # Fit model with bootstrap SEs
> fit2 = lavaan(pigmod, data=pigs, se = "bootstrap")
> summary(fit2) # I didn't ask for this explicitly, so no questions on it.
lavaan 0.6-11 ended normally after 120 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         9

  Number of observations                           114

Model Test User Model:

  Test statistic                                 0.087
  Degrees of freedom                                 1
  P-value (Chi-square)                           0.768

Parameter Estimates:

  Standard errors                            Bootstrap
  Number of requested bootstrap draws             1000
  Number of successful bootstrap draws            1000

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  Lbreeders =~
    nbreedersQ1       1.000
    nbreedersQ2       1.000
  Lbirth =~
    ngivebirthQ1      1.000
    ngivebirthQ2      1.000
```

```
Regressions:
                   Estimate  Std.Err  z-value  P(>|z|)
  Lbirth ~
    Lbredrs (beta)    0.757    0.071   10.721    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
 .nbreedersQ1 ~~
   .ngvbrQ1 (om12)  308.539   54.937    5.616    0.000
 .nbreedersQ2 ~~
   .ngvbrQ2 (om34)  101.406   37.255    2.722    0.006

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
    Lbredrs  (phi)  357.145   81.988    4.356    0.000
   .Lbirth   (psi)   33.634   11.319    2.972    0.003
   .nbrdrQ1 (omg1)  330.683   60.901    5.430    0.000
   .ngvbrQ1 (omg2)  321.255   52.548    6.114    0.000
   .nbrdrQ2 (omg3)  416.335   86.653    4.805    0.000
   .ngvbrQ2 (omg4)   93.000   30.053    3.095    0.002

Defined Parameters:
                   Estimate  Std.Err  z-value  P(>|z|)
    rel1              0.481    0.085    5.686    0.000
    rel2              0.538    0.082    6.537    0.000

>
> # Wald test of H0: omega1=omega3 and omega2=omega4
> That = coef(fit2); That
   beta     phi     psi  omega1  omega2  omega3  omega4 omega12 omega34
  0.757 357.145  33.634 330.683 321.255 416.335  93.000 308.539 101.406
> VV = vcov(fit2)
> LL = rbind(c(0, 0, 0, 1, 0,-1, 0, 0, 0),
+            c(0, 0, 0, 0, 1, 0,-1, 0, 0))
> Wtest(LL,That,VV)
           W           df      p-value
2.393441e+01 2.000000e+00 6.349046e-06
>
> # Follow-up
> # omega1 vs. omega3
> L13 = rbind(c(0, 0, 0, 1, 0,-1, 0, 0, 0))
> Wtest(L13,That,VV)
        W        df   p-value
0.7262899 1.0000000 0.3940884
> # omega2 vs. omega4
> L24 = rbind(c(0, 0, 0, 0, 1, 0,-1, 0, 0))
> Wtest(L24,That,VV)
           W           df      p-value
1.338262e+01 1.000000e+00 2.539661e-04
>
```