Name Jerry

Student Number

STA 431 Quiz 2

1. (3 points) Let $\mathbf{A}$ be a real, symmetric, positive definite matrix. Show that the eigenvalues of $\mathbf{A}$ are all strictly positive. Start with the definition $\mathbf{Ax} = \lambda\mathbf{x}$.

$$Ax = \lambda x \Rightarrow x^T A x = x^T \lambda x = \lambda x^T x = \lambda$$

Because A is positive definite,

$$0 < x^T A x = \lambda \quad \blacksquare$$

(Students can just use positive definite without writing the words.)

2. (3 points) Although eigen*vectors* are always non-zero, it is possible for an eigen*value* to equal zero. Let $\mathbf{A}$ be a square matrix, not necessarily symmetric, and let $(\lambda, \mathbf{x})$ be an (eigenvalue, eigenvector) pair with $\lambda = 0$. Show that $\mathbf{A}$ does not have an inverse.

$$Ax = \lambda x = 0 \cdot x = 0$$

Suppose A^{-1} exists. Then

$$A^{-1}Ax = A^{-1}0 = 0 \Rightarrow x = 0$$

But x cannot be zero because it is an eigenvector. This contradiction shows A^{-1} does not exist.

3. (4 points) Let the $p \times 1$ random vector $\mathbf{x}$ have expected value $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$, and let $\mathbf{A}$ be an $m \times p$ matrix of constants. Using the definition of a variance-covariance matrix on the formula sheet and familiar properties of expected value, derive the variance-covariance matrix of $\mathbf{Ax}$.

$$cov(Ax) = E\{(Ax - A\mu)(Ax - A\mu)^T\}$$

$$= E\{A(x-\mu)(A(x-\mu))^T\}$$

$$= E\{A(x-\mu)(x-\mu)^T A^T\}$$

$$= A\, E\{(x-\mu)(x-\mu)^T\} A^T$$

This step can be skipped

$$= A\, cov(x)\, A^T$$

$$= A \Sigma A^T$$

Or the student can stop at $A\,cov(x)\,A^T$ and skip the last step.