Name ______________________________

Student Number ______________________________

STA 431 Quiz 2

1. (3 points) Let $\mathbf{A}$ be a real, symmetric, positive definite matrix. Show that the eigenvalues of $\mathbf{A}$ are all strictly positive. Start with the definition $\mathbf{Ax} = \lambda\mathbf{x}$.

2. (3 points) Although eigen*vectors* are always non-zero, it is possible for an eigen*value* to equal zero. Let $\mathbf{A}$ be a square matrix, not necessarily symmetric, and let $(\lambda, \mathbf{x})$ be an (eigenvalue, eigenvector) pair with $\lambda = 0$. Show that $\mathbf{A}$ does not have an inverse.

3. (4 points) Let the $p \times 1$ random vector $\mathbf{x}$ have expected value $\boldsymbol{\mu}$ and variance-covariance matrix $\boldsymbol{\Sigma}$, and let $\mathbf{A}$ be an $m \times p$ matrix of constants. Using the definition of a variance-covariance matrix on the formula sheet and familiar properties of expected value, derive the variance-covariance matrix of $\mathbf{Ax}$.