Name Jerry

Student Number

STA 431 Quiz 1

Circle

1. (5 points) Let the random variable x have expected value μ_x, let the random variable y have expected value μ_y, and let a be a non-zero constant. ~~Choose~~ one the of following statements and prove it, *using the definition of covariance* from the formula sheet.

   $Cov(ax, y) = a^2 Cov(x, y)$, $Cov(ax, y) = aCov(x, y)$, $Cov(ax, y) = Cov(x, y)$, $Cov(ax, y) = 0$

$$Cov(ax, y) = E\{(ax - a\mu_x)(y - \mu_y)\}$$
$$= E\{a(x - \mu_x)(y - \mu_y)\}$$
$$= a E\{(x - \mu_x)(y - \mu_y)\}$$
$$= a\, Cov(x, y)$$

2. (5 points) Let

$$y_1 = \alpha_1 + \beta_1 x + \epsilon_1$$
$$y_2 = \alpha_2 + \beta_2 x + \epsilon_2,$$

where $E(x) = \mu$, $Var(x) = \sigma_x^2$, $E(\epsilon_1) = E(\epsilon_2) = 0$, $Var(\epsilon_1) = \sigma_1^2$ and $Var(\epsilon_2) = \sigma_2^2$. The random variables x, ϵ_1 and ϵ_2 are independent. Using anything you wish from the formula sheet, calculate $Cov(y_1, y_2)$.

$$Cov(y_1, y_2) = Cov((\alpha_1 + \beta_1 x + \varepsilon_1)(\alpha_2 + \beta_2 x + \varepsilon_2))$$
$$= \beta_1 \beta_2 Cov(x, x) + \beta_1 Cov(x, \varepsilon_2) + \beta_2 Cov(x, \varepsilon_1) + Cov(\varepsilon_1, \varepsilon_2)$$
$$= \beta_1 \beta_2 Var(x) + 0 + 0 + 0$$
$$= \beta_1 \beta_2 \sigma_x^2$$

(It's okay to skip a step)

No marks off for not circling.