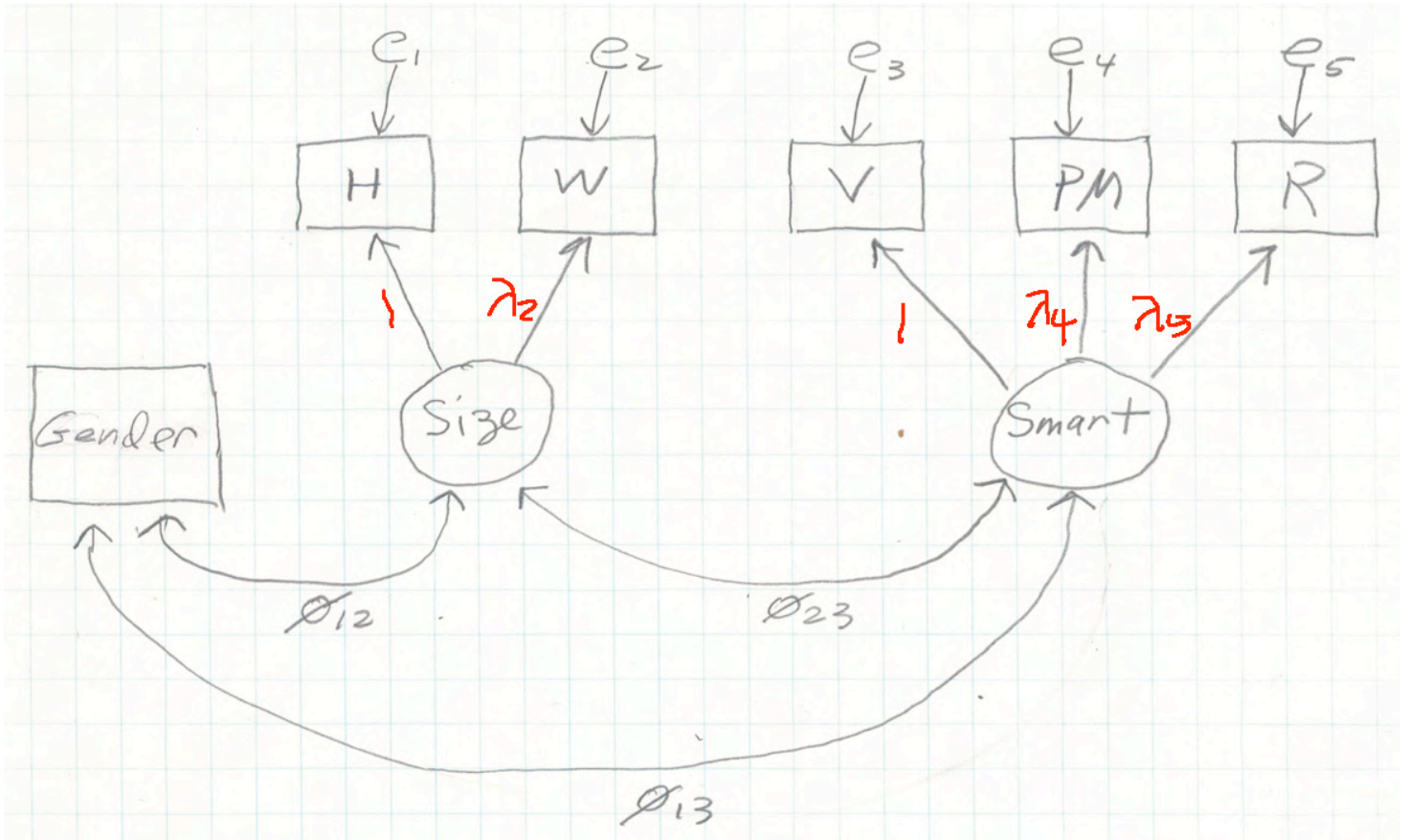


Confirmatory Factor Analysis of the Body-mind Data*

```
> # install.packages("lavaan", dependencies = TRUE) # Only need to do this once
> library(lavaan)
This is lavaan 0.6-15
lavaan is FREE software! Please report any bugs.
>
> rm(list=ls())
> bodymind =
read.table("http://www.utstat.toronto.edu/~brunner/openSEM/data/bodymind.data.txt")
> head(bodymind)
  sex progmat reason verbal headlmg headbrd headcir bizygz weight height
1   M     108    128    136    182     162    553    140    144    1769
2   F      81    110     94    192     156    571    143    144    1633
3   F     110    134    132    186     145    549    131    135    1672
4   F      95     88     83    189     139    536    124    109    1700
5   M      83     94    100    180     163    549    141    124    1679
6   M     105     77     92    195     148    560    134    126    1651
>
> # Need a numeric dummy variable for sex/gender
> n = dim(bodymind)[1]; n
[1] 80
> bodymind = within(bodymind,{
+   Sex = numeric(n)
+   Sex[sex == "F"] = 1 # Makes 0=M, 1=F
+ }) # End within bodymind
> head(bodymind)
  sex progmat reason verbal headlmg headbrd headcir bizygz weight height Sex
1   M     108    128    136    182     162    553    140    144    1769  0
2   F      81    110     94    192     156    571    143    144    1633  1
3   F     110    134    132    186     145    549    131    135    1672  1
4   F      95     88     83    189     139    536    124    109    1700  1
5   M      83     94    100    180     163    549    141    124    1679  0
6   M     105     77     92    195     148    560    134    126    1651  0
```

*This handout was prepared by Jerry Brunner, Department of Statistical Sciences, University of Toronto. It is licensed under a Creative Commons Attribution - ShareAlike 3.0 Unported License. Use any part of it as you like and share the result freely. The OpenOffice.org document is available from the course website:

<http://www.utstat.toronto.edu/brunner/oldclass/431s23>



```

> # Observe that the parameters of the sub-model with Sex and size are identifiable.
>
> # In mod1, name all the parameters and use the lavaan function.
> mod1 = "#####
+ # Measurement model
+ # -----
+ size =~ 1.0*height + lambda2*weight
+ smart =~ 1.0*verbal + lambda4*progmatt + lambda5*reason
+ # Variances and covariances
+ # -----
+ Sex =~ phi11*Sex + phi12*size + phi13*smart
+ size =~ phi22*size + phi23*smart
+ smart =~ phi33*smart
+ height =~ omega1*height # Var(e1) = omega1
+ weight =~ omega2*weight # Var(e2) = omega2
+ verbal =~ omega3*verbal # Var(e3) = omega3
+ progmatt =~ omega4*progmatt # Var(e4) = omega4
+ reason =~ omega5*reason # Var(e5) = omega5
+ #####
+ " # End of mod1
>

```

```

> fit1 = lavaan(mod1, data=bodymind)
Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate

```

```

> varTable(fit1)
  name idx nobs   type exo user   mean   var nlev lnam
1 height  10   80 numeric   0   0 1651.862 6084.019   0
2 weight   9   80 numeric   0   0  124.100  479.357   0
3 verbal   4   80 numeric   0   0   99.625  637.402   0
4 progmat  2   80 numeric   0   0   98.700   98.719   0
5 reason   3   80 numeric   0   0  100.875  272.465   0
6     Sex  11   80 numeric   0   0    0.537   0.252   0

> # It may be okay, but re-express height in metres
> bodymind = within(bodymind,{height = height/10})

> fit1 = lavaan(mod1, data=bodymind)
Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate

```

```

> summary(fit1)
lavaan 0.6.15 ended normally after 205 iterations

Estimator ML
Optimization method NLMINB
Number of model parameters 14

Number of observations 80

Model Test User Model:

Test statistic 26.079
Degrees of freedom 7
P-value (Chi-square) 0.000

Parameter Estimates:

Standard errors Standard
Information Expected
Information saturated (h1) model Structured

Latent Variables:
      Estimate Std.Err z-value P(>|z|)
size =~
  height      1.000
  weight (lmb2) 2.138    0.467    4.575    0.000
smart =~
  verbal      1.000
  progmat (lmb4) 0.283    0.054    5.259    0.000
  reason (lmb5) 0.707    0.106    6.658    0.000

Covariances:
      Estimate Std.Err z-value P(>|z|)
size ~~
  Sex (ph12) -1.839    0.477   -3.858    0.000
smart ~~
  Sex (ph13)  0.856    1.244    0.689    0.491
size ~~
  smart (ph23) 40.583   19.846    2.045    0.041

Variances:
      Estimate Std.Err z-value P(>|z|)
Sex (ph11)  0.249    0.039    6.325    0.000
size (ph22) 47.220   12.684    3.723    0.000
smart (ph33) 427.111  107.773    3.963    0.000
.height (omg1) 12.860    8.883    1.448    0.148
.weight (omg2) 257.430   56.747    4.536    0.000
.verbal (omg3) 202.323   61.294    3.301    0.001
.progmat (omg4) 63.352   10.996    5.761    0.000
.reason (omg5) 55.454   27.471    2.019    0.044

> # Variance of Sex is really p(1-p) -- Bernoulli.
> # Estimates are well behaved. I think it's okay.
>
> # By the way, the model does not fit. This is an unpleasant surprise, but these are real
data.

```

```

> # Fit the model more easily with cfa. Sex is included in the last line.
> mod2 = "size =~ height + weight
+       smart =~ verbal + progmatt + reason
+       Sex =~ size + smart" # Last line is unusual.
>
> fit2 = cfa(mod2, data=bodymind)
Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate
> summary(fit2)
lavaan 0.6.15 ended normally after 205 iterations

Estimator                               ML
Optimization method                     NLMINB
Number of model parameters              14

Number of observations                   80

Model Test User Model:

Test statistic                           26.079
Degrees of freedom                        7
P-value (Chi-square)                     0.000

Parameter Estimates:

Standard errors                           Standard
Information                               Expected
Information saturated (h1) model          Structured

Latent Variables:

Estimate  Std.Err  z-value  P(>|z|)
size =~
  height      1.000
  weight     2.138    0.467    4.575    0.000
smart =~
  verbal      1.000
  progmatt   0.283    0.054    5.259    0.000
  reason     0.707    0.106    6.658    0.000

Covariances:

Estimate  Std.Err  z-value  P(>|z|)
size ~~
  Sex      -1.839    0.477   -3.858    0.000
smart ~~
  Sex       0.856    1.244    0.689    0.491
size ~~
  smart    40.583   19.846    2.045    0.041

Variances:

Estimate  Std.Err  z-value  P(>|z|)
.height   12.860    8.883    1.448    0.148
.weight   257.430   56.747    4.536    0.000
.verbal   202.323   61.294    3.301    0.001
.progmatt 63.352   10.996    5.761    0.000
.reason   55.454   27.471    2.019    0.044
Sex        0.249    0.039    6.325    0.000
size      47.220   12.684    3.723    0.000
smart    427.111  107.773    3.963    0.000

>
> # Parameter estimates match perfectly.

```

```
> # "Standardized parameter estimates" (Adds two columns ...)
> summary(fit2, standardized=TRUE)
```

lavaan 0.6.15 ended normally after 205 iterations

```
Estimator ML
Optimization method NLMINB
Number of model parameters 14
```

```
Number of observations 80
```

Model Test User Model:

```
Test statistic 26.079
Degrees of freedom 7
P-value (Chi-square) 0.000
```

Parameter Estimates:

```
Standard errors Standard
Information Expected
Information saturated (h1) model Structured
```

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
size =~						
height	1.000				6.872	0.887
weight	2.138	0.467	4.575	0.000	14.695	0.675
smart =~						
verbal	1.000				20.667	0.824
progmatt	0.283	0.054	5.259	0.000	5.842	0.592
reason	0.707	0.106	6.658	0.000	14.615	0.891

Covariances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
size ~~						
Sex	-1.839	0.477	-3.858	0.000	-0.268	-0.537
smart ~~						
Sex	0.856	1.244	0.689	0.491	0.041	0.083
size ~~						
smart	40.583	19.846	2.045	0.041	0.286	0.286

Variances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
.height	12.860	8.883	1.448	0.148	12.860	0.214
.weight	257.430	56.747	4.536	0.000	257.430	0.544
.verbal	202.323	61.294	3.301	0.001	202.323	0.321
.progmatt	63.352	10.996	5.761	0.000	63.352	0.650
.reason	55.454	27.471	2.019	0.044	55.454	0.206
Sex	0.249	0.039	6.325	0.000	0.249	1.000
size	47.220	12.684	3.723	0.000	1.000	1.000
smart	427.111	107.773	3.963	0.000	1.000	1.000

```
> # Note: Calculation of estimates for the standardized models is described in the text.
```

```

> # The only problem with Std.lv and Std.all is that you don't get standard errors.
>
> Bmdat = bodymind[,c(2:4, 9:11)]
> kor = cor(Bmdat); round(kor,3)
      progmatt reason verbal weight height Sex
progmatt  1.000  0.514  0.539  0.132  0.197 -0.257
reason    0.514  1.000  0.728  0.171  0.207  0.186
verbal    0.539  0.728  1.000  0.236  0.199 -0.011
weight    0.132  0.171  0.236  1.000  0.599 -0.346
height    0.197  0.207  0.199  0.599  1.000 -0.481
Sex       -0.257  0.186 -0.011 -0.346 -0.481  1.000
> fit3 = cfa(mod2, sample.cov=kor, sample.nobs=80, std.lv=TRUE, sample.cov.rescale=FALSE)
> summary(fit3, standardized=TRUE)

```

lavaan 0.6.15 ended normally after 21 iterations

```

Estimator ML
Optimization method NLMINB
Number of model parameters 14

Number of observations 80

```

Model Test User Model:

```

Test statistic 26.079
Degrees of freedom 7
P-value (Chi-square) 0.000

```

Parameter Estimates:

```

Standard errors Standard
Information Expected
Information saturated (h1) model Structured

```

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
size =~						
height	0.887	0.119	7.446	0.000	0.887	0.887
weight	0.675	0.116	5.819	0.000	0.675	0.675
smart =~						
verbal	0.824	0.104	7.926	0.000	0.824	0.824
progmatt	0.592	0.109	5.434	0.000	0.592	0.592
reason	0.891	0.102	8.701	0.000	0.891	0.891

Covariances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
size ~~						
Sex	-0.537	0.117	-4.568	0.000	-0.537	-0.537
smart ~~						
Sex	0.083	0.120	0.693	0.488	0.083	0.083
size ~~						
smart	0.286	0.125	2.285	0.022	0.286	0.286

Variances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
.height	0.214	0.148	1.448	0.148	0.214	0.214
.weight	0.544	0.120	4.536	0.000	0.544	0.544
.verbal	0.321	0.097	3.301	0.001	0.321	0.321
.progmatt	0.650	0.113	5.761	0.000	0.650	0.650
.reason	0.206	0.102	2.019	0.044	0.206	0.206
Sex	1.000	0.158	6.325	0.000	1.000	1.000
size	1.000				1.000	1.000
smart	1.000				1.000	1.000

```

> # Perfect match, no complaints
> # Note how std.lv=TRUE wisely un-did the setting of factor loadings to one.

```

```

> # Why is the model not fitting?
> # Look at the correlation matrix again.
> round(kor,3)

      progmatt reason verbal weight height Sex
progmatt 1.000 0.514 0.539 0.132 0.197 -0.257
reason 0.514 1.000 0.728 0.171 0.207 0.186
verbal 0.539 0.728 1.000 0.236 0.199 -0.011
weight 0.132 0.171 0.236 1.000 0.599 -0.346
height 0.197 0.207 0.199 0.599 1.000 -0.481
Sex -0.257 0.186 -0.011 -0.346 -0.481 1.000

> # Residuals are model-implied covariances (correlations),
> # minus observed covariances (correlations).
>
> lavResiduals(fit2) # cor by default

```

```

$type
[1] "cor.bentler"

```

```

$cov
      height weight verbal progmatt reason Sex
height 0.000
weight 0.000 0.000
verbal -0.009 0.077 0.000
progmatt 0.047 0.018 0.051 0.000
reason -0.019 -0.001 -0.006 -0.013 0.000
Sex -0.005 0.016 -0.079 -0.306 0.111 0.000

```

```

$cov.z
      height weight verbal progmatt reason Sex
height 0.000
weight 0.000 0.000
verbal -0.188 1.027 0.000
progmatt 0.564 0.205 2.151 0.000
reason -0.519 -0.016 -2.361 -1.048 0.000
Sex -0.442 0.442 -1.493 -3.350 3.019 0.000

```

```

$summary
      cov
srmmr 0.077
srmmr.se 0.015
srmmr.exactfit.z 2.184
srmmr.exactfit.pvalue 0.014
usrmmr 0.065
usrmmr.se 0.025
usrmmr.ci.lower 0.024
usrmmr.ci.upper 0.105
usrmmr.closefit.h0.value 0.050
usrmmr.closefit.z 0.594
usrmmr.closefit.pvalue 0.276

```

```

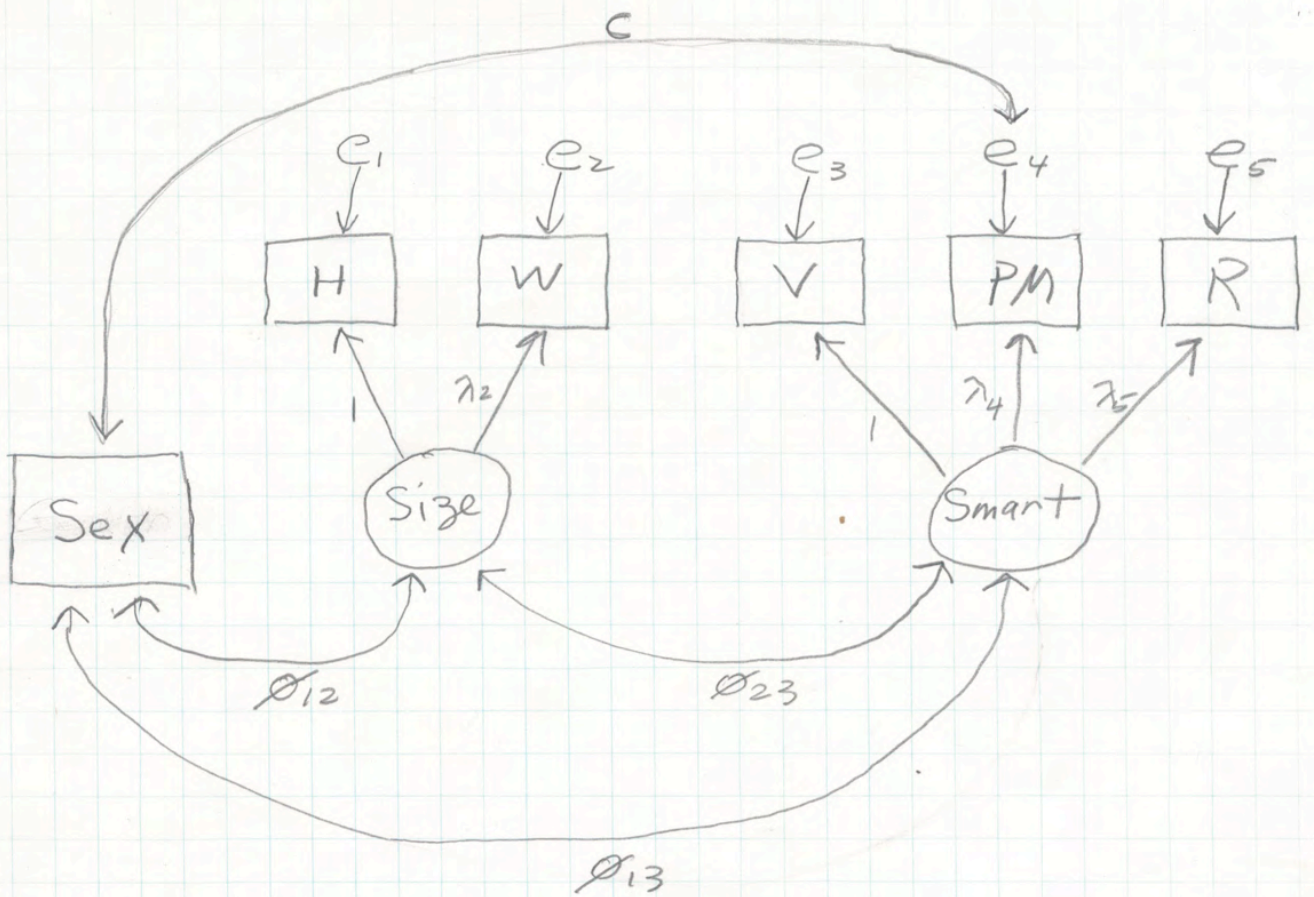
> fitted(fit2) # Sigma(thetahat)
$cov
      height weight verbal progmatt reason Sex
height 60.080
weight 100.978 473.365
verbal 40.583 86.785 629.434
progmatt 11.473 24.533 120.741 97.485
reason 28.700 61.373 302.048 85.387 269.059
Sex -1.839 -3.932 0.856 0.242 0.606 0.249

```

```

> # How about an additional link between Sex and progmatt?

```

	Sex	H	W	V	PM	R
Sex	ϕ_{11}	ϕ_{12}	$\lambda_2 \phi_{12}$	ϕ_{13}	$\lambda_4 \phi_{13} + c$	$\lambda_5 \phi_{13}$
H		$\phi_{22} + w_1$	$\lambda_2 \phi_{22}$	ϕ_{23}	$\lambda_4 \phi_{23}$	$\lambda_5 \phi_{23}$
W			$\lambda_2^2 \phi_{22} + w_2$	$\lambda_2 \phi_{23}$	$\lambda_2 \lambda_4 \phi_{23}$	$\lambda_2 \lambda_5 \phi_{23}$
V				$\phi_{33} + w_3$	$\lambda_4 \phi_{33}$	$\lambda_5 \phi_{33}$
PM					$\lambda_4^2 \phi_{33} + w_4$	$\lambda_4 \lambda_5 \phi_{33}$
R						$\lambda_5^2 \phi_{33} + w_5$

```

> mod3 = "size =~ height + weight
+       smart =~ verbal + progmat + reason
+       Sex ~~ size + smart + progmat"
> fit4 = cfa(mod3, data=bodymind)
Error in tmp[cbind(REP$row[idx], REP$col[idx])] <- lavpartable$free[idx] :
  NAs are not allowed in subscripted assignments
In addition: Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
  lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate

> # That was ugly and strange. Try fuller model specification
>
>
> mod4 = "#####
+       # Measurement model
+       # -----
+       size =~ 1.0*height + lambda2*weight
+       smart =~ 1.0*verbal + lambda4*progmat + lambda5*reason
+       # Variances and covariances
+       # -----
+       Sex ~~ phi11*Sex + phi12*size + phi13*smart
+               size ~~ phi22*size + phi23*smart
+               smart ~~ phi33*smart
+       Sex ~~ c*progmat # Covariance with error term
+       height ~~ omega1*height # Var(e1) = omega1
+       weight  ~~ omega2*weight # Var(e2) = omega2
+       verbal  ~~ omega3*verbal # Var(e3) = omega3
+       progmat ~~ omega4*progmat # Var(e4) = omega4
+       reason  ~~ omega5*reason # Var(e5) = omega5
+       #####
+       " # End of mod4
> fit4 = lavaan(mod4, data=bodymind)
Error in tmp[cbind(REP$row[idx], REP$col[idx])] <- lavpartable$free[idx] :
  NAs are not allowed in subscripted assignments
In addition: Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
  lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate
> # Tried other things ...

# Post to the group and display that

```

```

> # Make the link from smart to progmata a "regression."
>
> mod5 = "#####
+ # Regressions
+ # -----
+ progmata ~ lambda4 * smart
+ # Measurement model
+ # -----
+ size =~ 1.0*height + lambda2*weight
+ smart =~ 1.0*verbal + lambda5*reason
+ # Variances and covariances
+ # -----
+ Sex ~~ phi11*Sex + phi12*size + phi13*smart
+ size ~~ phi22*size + phi23*smart
+ smart ~~ phi33*smart
+ Sex ~~ c*progmata # Covariance with error term -- an epsilon this time.
+ height ~~ omega1*height # Var(e1) = omega1
+ weight ~~ omega2*weight # Var(e2) = omega2
+ verbal ~~ omega3*verbal # Var(e3) = omega3
+ progmata ~~ omega4*progmata # Var(e4) = omega4
+ reason ~~ omega5*reason # Var(e5) = omega5
+ #####
+ " # End of mod5
> fit5 = lavaan(mod5, data=bodymind)
Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate
> summary(fit5)
lavaan 0.6-12 ended normally after 188 iterations

Estimator ML
Optimization method NLMINB
Number of model parameters 15

Number of observations 80

Model Test User Model:

Test statistic 8.193
Degrees of freedom 6
P-value (Chi-square) 0.224

Parameter Estimates:

Standard errors Standard
Information Expected
Information saturated (h1) model Structured

Latent Variables:
Estimate Std.Err z-value P(>|z|)
size =~
height 1.000
weight (lmb2) 21.946 4.504 4.873 0.000
smart =~
verbal 1.000
reason (lmb5) 0.766 0.103 7.465 0.000

Regressions:
Estimate Std.Err z-value P(>|z|)
progmata ~
smart (lmb4) 0.293 0.056 5.249 0.000

```

Covariances:					
		Estimate	Std.Err	z-value	P(> z)
size	~~				
Sex	(ph12)	-0.174	0.044	-3.958	0.000
smart	~~				
Sex	(ph13)	1.471	1.180	1.247	0.212
size	~~				
smart	(ph23)	3.749	1.869	2.006	0.045
.progmatt	~~				
Sex	(c)	-1.569	0.421	-3.727	0.000

Variances:					
		Estimate	Std.Err	z-value	P(> z)
Sex	(ph11)	0.242	0.037	6.591	0.000
size	(ph22)	0.460	0.121	3.815	0.000
smart	(ph33)	390.685	99.636	3.921	0.000
.height	(omg1)	0.141	0.081	1.743	0.081
.weight	(omg2)	251.756	54.598	4.611	0.000
.verbal	(omg3)	238.750	53.597	4.455	0.000
.progmatt	(omg4)	63.844	10.980	5.815	0.000
.reason	(omg5)	40.063	23.112	1.733	0.083

```

>
> # Add functions for the CORRELATIONS between factors
>
> mod6 = "#####
+ # Regressions
+ # -----
+ progmat ~ lambda4 * smart
+ # Measurement model
+ # -----
+ size =~ 1.0*height + lambda2*weight
+ smart =~ 1.0*verbal + lambda5*reason
+ # Variances and covariances
+ # -----
+ Sex ~~ phil1*Sex + phil2*size + phil3*smart
+ size ~~ phi22*size + phi23*smart
+ smart ~~ phi33*smart
+ Sex ~~ c*progmat # Covariance with error term -- an epsilon this time.
+ height ~~ omega1*height # Var(e1) = omega1
+ weight ~~ omega2*weight # Var(e2) = omega2
+ verbal ~~ omega3*verbal # Var(e3) = omega3
+ progmat ~~ omega4*progmat # Var(e4) = omega4
+ reason ~~ omega5*reason # Var(e5) = omega5
+ # Correlations between factors
+ # -----
+ corr12 := phi12/sqrt(phi11*phi22)
+ corr13 := phi13/sqrt(phi11*phi33)
+ corr23 := phi23/sqrt(phi22*phi33)
+ #####
+ " # End of mod6
> fit6 = lavaan(mod6, data=bodymind)
Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate

```

```
> summary(fit6, standardized=TRUE)
lavaan 0.6-12 ended normally after 188 iterations
```

```
Estimator ML
Optimization method NLMINB
Number of model parameters 15

Number of observations 80
```

Model Test User Model:

```
Test statistic 8.193
Degrees of freedom 6
P-value (Chi-square) 0.224
```

Parameter Estimates:

```
Standard errors Standard
Information Expected
Information saturated (h1) model Structured
```

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
size =~						
height	1.000				0.678	0.875
weight (lmb2)	21.946	4.504	4.873	0.000	14.887	0.684
smart =~						
verbal	1.000				19.766	0.788
reason (lmb5)	0.766	0.103	7.465	0.000	15.133	0.923

Regressions:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
progmatt ~						
smart (lmb4)	0.293	0.056	5.249	0.000	5.800	0.587

Covariances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
size ~~						
Sex (ph12)	-0.174	0.044	-3.958	0.000	-0.256	-0.520
smart ~~						
Sex (ph13)	1.471	1.180	1.247	0.212	0.074	0.151
size ~~						
smart (ph23)	3.749	1.869	2.006	0.045	0.280	0.280
.progmatt ~~						
Sex (c)	-1.569	0.421	-3.727	0.000	-1.569	-0.399

Variances:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
Sex (ph11)	0.242	0.037	6.591	0.000	0.242	1.000
size (ph22)	0.460	0.121	3.815	0.000	1.000	1.000
smart (ph33)	390.685	99.636	3.921	0.000	1.000	1.000
.height (omg1)	0.141	0.081	1.743	0.081	0.141	0.234
.weight (omg2)	251.756	54.598	4.611	0.000	251.756	0.532
.verbal (omg3)	238.750	53.597	4.455	0.000	238.750	0.379
.progmatt (omg4)	63.844	10.980	5.815	0.000	63.844	0.655
.reason (omg5)	40.063	23.112	1.733	0.083	40.063	0.149

Defined Parameters:

	Estimate	Std.Err	z-value	P(> z)	Std.lv	Std.all
corr12	-0.520	0.092	-5.647	0.000	-0.520	-0.520
corr13	0.151	0.117	1.296	0.195	0.151	0.151
corr23	0.280	0.125	2.243	0.025	0.280	0.280

```

> # Bring in the head size variables. No need for symbols.
>
> mod7 = "#####
+ # Regressions
+ # -----
+ progm4 ~ lambda4 * smart
+   headlng ~ Sex + size + smart
+   headbrd ~ Sex + size + smart
+   headcir ~ Sex + size + smart
+   bizyg   ~ Sex + size + smart
+ # Measurement model
+ # -----
+ size  =~ 1.0*height + lambda2*weight
+ smart =~ 1.0*verbal + lambda5*reason
+ # Variances and covariances
+ # -----
+ Sex  ~~ phil1*Sex + phil2*size + phil3*smart
+      size  ~~ phi22*size + phi23*smart
+      smart ~~ phi33*smart
+ Sex  ~~ c*progm4 # Covariance with error term -- an epsilon this time.
+ height  ~~ omega1*height # Var(e1) = omega1
+ weight  ~~ omega2*weight  # Var(e2) = omega2
+ verbal  ~~ omega3*verbal  # Var(e3) = omega3
+ progm4  ~~ omega4*progm4  # Var(e4) = omega4
+ reason  ~~ omega5*reason  # Var(e5) = omega5
+ # Variances and covariances between error terms of head variables
+ # -----
+ headlng ~~ headlng + headbrd + headcir + bizyg
+          headbrd  ~~ headbrd + headcir + bizyg
+          headcir  ~~ headcir + bizyg
+          bizyg    ~~ bizyg
+
+ # Correlations between factors
+ # -----
+ corr12 := phi12/sqrt(phi11*phi22)
+ corr13 := phi13/sqrt(phi11*phi33)
+ corr23 := phi23/sqrt(phi22*phi33)
+ #####
+ " # End of mod7
>
> fit8 = lavaan(mod7, data=bodymind); summary(fit8)
Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate

```

lavaan 0.6-12 ended normally after 536 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	37
Number of observations	80

Model Test User Model:

Test statistic	20.489
Degrees of freedom	18
P-value (Chi-square)	0.306

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
size =~				
height	1.000			
weight (lmb2)	27.434	3.975	6.902	0.000
smart =~				
verbal	1.000			
reason (lmb5)	0.743	0.092	8.097	0.000

Regressions:

	Estimate	Std.Err	z-value	P(> z)
progmatt ~				
smart (lmb4)	0.295	0.055	5.378	0.000
headlng ~				
Sex	-2.884	1.835	-1.572	0.116
size	6.260	1.866	3.355	0.001
smart	0.054	0.041	1.320	0.187
headbrd ~				
Sex	-5.085	1.712	-2.970	0.003
size	2.909	1.662	1.750	0.080
smart	0.015	0.038	0.397	0.691
headcir ~				
Sex	-3.178	4.064	-0.782	0.434
size	19.560	4.366	4.480	0.000
smart	0.176	0.090	1.953	0.051
bizygg ~				
Sex	-1.682	1.313	-1.281	0.200
size	6.281	1.410	4.456	0.000
smart	0.045	0.029	1.536	0.125

Covariances:

	Estimate	Std.Err	z-value	P(> z)
size ~~				
Sex (ph12)	-0.157	0.041	-3.806	0.000
smart ~~				
Sex (ph13)	1.413	1.205	1.172	0.241
size ~~				
smart (ph23)	3.617	1.775	2.038	0.042
.progmatt ~~				
Sex (c)	-1.595	0.421	-3.786	0.000
.headlng ~~				
.headbrd	-5.350	3.543	-1.510	0.131
.headcir	35.189	10.038	3.506	0.000
.bizygg	-3.299	2.687	-1.228	0.220

```
.headbrd ~~
  .headcir      24.512    8.439    2.905    0.004
  .bizyg        10.486    2.855    3.672    0.000
.headcir ~~
  .bizyg         6.666    6.484    1.028    0.304
```

Variances:

	Estimate	Std.Err	z-value	P(> z)
Sex (ph11)	0.244	0.037	6.604	0.000
size (ph22)	0.369	0.095	3.904	0.000
smart (ph33)	401.867	98.771	4.069	0.000
.height (omg1)	0.232	0.051	4.550	0.000
.weight (omg2)	195.666	40.925	4.781	0.000
.verbal (omg3)	227.567	49.401	4.607	0.000
.progmatt (omg4)	62.441	10.801	5.781	0.000
.reason (omg5)	47.424	19.830	2.391	0.017
.headlng	28.282	5.284	5.353	0.000
.headbrd	28.755	4.709	6.107	0.000
.headcir	108.007	25.495	4.236	0.000
.bizyg	11.432	2.656	4.305	0.000

Defined Parameters:

	Estimate	Std.Err	z-value	P(> z)
corr12	-0.522	0.092	-5.707	0.000
corr13	0.143	0.118	1.210	0.226
corr23	0.297	0.129	2.309	0.021

```
>
> # I am not really comfortable with arrows coming from Sex.
>
> mod8 = "#####
+ # Regressions
+ # -----
+ progmat ~ lambda4 * smart
+ headlng ~ size + smart
+ headbrd ~ size + smart
+ headcir ~ size + smart
+ bizyg ~ size + smart
+ # Measurement model
+ # -----
+ size =~ 1.0*height + lambda2*weight
+ smart =~ 1.0*verbal + lambda5*reason
+ # Variances and covariances
+ # -----
+ Sex ~~ phil1*Sex + phil2*size + phil3*smart
+ size ~~ phi22*size + phi23*smart
+ smart ~~ phi33*smart
+ Sex ~~ c*progmat # Covariance with error term -- an epsilon this time.
+ height ~~ omega1*height # Var(e1) = omega1
+ weight ~~ omega2*weight # Var(e2) = omega2
+ verbal ~~ omega3*verbal # Var(e3) = omega3
+ progmat ~~ omega4*progmat # Var(e4) = omega4
+ reason ~~ omega5*reason # Var(e5) = omega5
+ # Variances and covariances between error terms of head variables
+ # -----
+ headlng ~~ headlng + headbrd + headcir + bizyg
+ headbrd ~~ headbrd + headcir + bizyg
+ headcir ~~ headcir + bizyg
+ bizyg ~~ bizyg
+ # Correlations between factors
+ # -----
+ corr12 := phi12/sqrt(phi11*phi22)
+ corr13 := phi13/sqrt(phi11*phi33)
+ corr23 := phi23/sqrt(phi22*phi33)
+ #####
+ " # End of mod8
>
```



```
> fit9 = lavaan(mod8, data=bodymind); summary(fit9)
Warning message:
In lav_data_full(data = data, group = group, cluster = cluster, :
lavaan WARNING: some observed variances are (at least) a factor 1000 times larger than
others; use varTable(fit) to investigate
```

lavaan 0.6-12 ended normally after 551 iterations

```
Estimator ML
Optimization method NLMINB
Number of model parameters 33

Number of observations 80
```

Model Test User Model:

```
Test statistic 43.333
Degrees of freedom 22
P-value (Chi-square) 0.004
```

Parameter Estimates:

```
Standard errors Standard
Information Expected
Information saturated (h1) model Structured
```

Latent Variables:

	Estimate	Std.Err	z-value	P(> z)
size =~				
height	1.000			
weight (lmb2)	25.844	4.074	6.344	0.000
smart =~				
verbal	1.000			
reason (lmb5)	0.747	0.092	8.135	0.000

Regressions:

	Estimate	Std.Err	z-value	P(> z)
progmatt ~				
smart (lmb4)	0.295	0.056	5.242	0.000
headlng ~				
size	8.462	1.536	5.508	0.000
smart	0.019	0.039	0.481	0.631
headbrd ~				
size	6.991	1.445	4.839	0.000
smart	-0.049	0.038	-1.289	0.197
headcir ~				
size	22.088	3.476	6.354	0.000
smart	0.133	0.086	1.542	0.123
bizyg ~				
size	7.768	1.155	6.725	0.000
smart	0.020	0.028	0.724	0.469

Covariances:

	Estimate	Std.Err	z-value	P(> z)
size ~~				
Sex (ph12)	-0.173	0.042	-4.117	0.000
smart ~~				
Sex (ph13)	1.594	1.186	1.344	0.179
size ~~				
smart (ph23)	3.988	1.792	2.226	0.026
.progmatt ~~				
Sex (c)	-1.544	0.383	-4.034	0.000
.headlng ~~				
.headbrd	-7.156	3.754	-1.906	0.057
.headcir	32.279	10.132	3.186	0.001
.bizyg	-4.772	2.699	-1.768	0.077

.headbrd	~~				
.headcir		18.246	8.838	2.065	0.039
.bizyg		8.147	2.974	2.739	0.006
.headcir	~~				
.bizyg		3.825	6.310	0.606	0.544

Variances:

		Estimate	Std.Err	z-value	P(> z)
Sex	(ph11)	0.239	0.036	6.721	0.000
size	(ph22)	0.354	0.091	3.871	0.000
smart	(ph33)	392.165	97.483	4.023	0.000
.height	(omg1)	0.247	0.049	5.050	0.000
.weight	(omg2)	236.832	43.215	5.480	0.000
.verbal	(omg3)	237.270	49.122	4.830	0.000
.progmatt	(omg4)	63.394	10.936	5.797	0.000
.reason	(omg5)	50.475	18.836	2.680	0.007
.headlng		27.027	5.434	4.974	0.000
.headbrd		27.757	5.103	5.439	0.000
.headcir		104.399	25.110	4.158	0.000
.bizyg		9.930	2.590	3.833	0.000

Defined Parameters:

	Estimate	Std.Err	z-value	P(> z)
corr12	-0.595	0.080	-7.413	0.000
corr13	0.165	0.118	1.400	0.161
corr23	0.338	0.130	2.609	0.009

>
>

> anova(fit8,fit9) # Should be 4 df

Chi-Squared Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	Df diff	Pr(>Chisq)
fit8	18	4848.6	4936.8	20.489			
fit9	22	4863.5	4942.1	43.333	22.844	4	0.0001361 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1