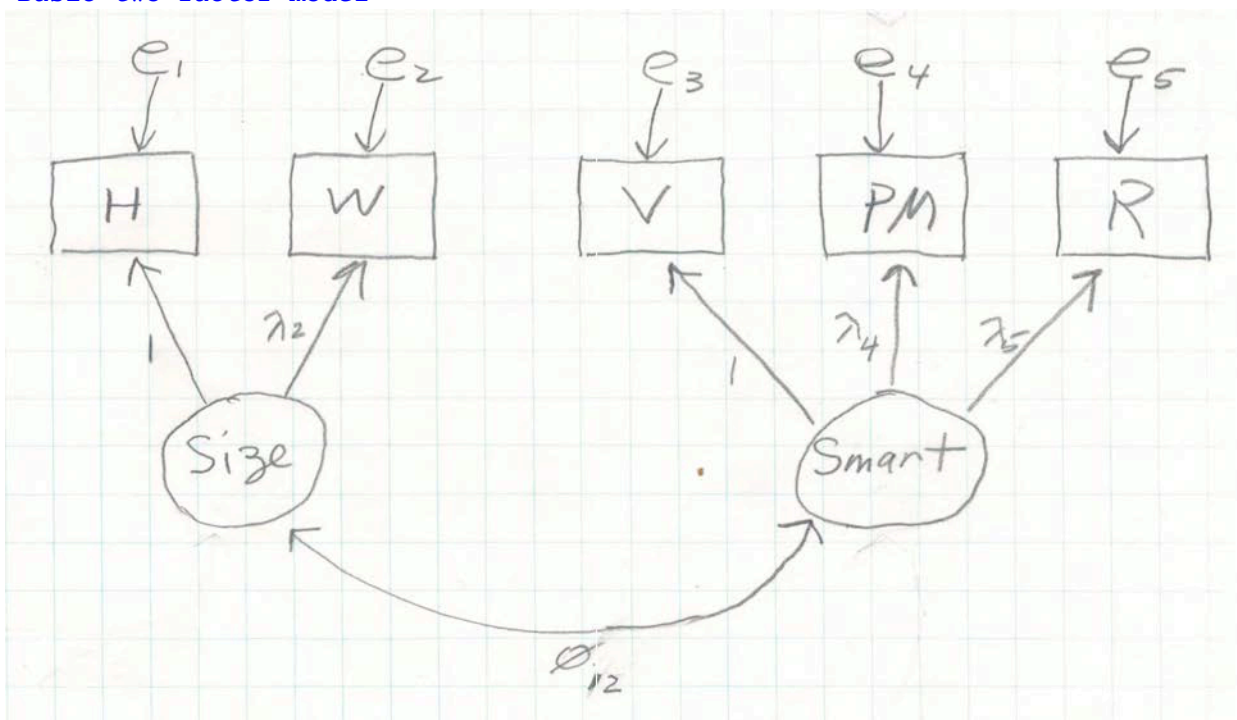


# Multi-group analysis with lavaan\*

```
> # install.packages("lavaan", dependencies = TRUE) # Only need to do this once
> library(lavaan)
This is lavaan 0.6-15
lavaan is FREE software! Please report any bugs.
>
> rm(list=ls())
> bodymind =
read.table("http://www.utstat.toronto.edu/~brunner/openSEM/data/bodymind.data.txt")
> head(bodymind)
  sex progmat reason verbal heading headbrd headcir bizyg weight height
1  M     108    128    136    182     162    553    140    144   1769
2  F      81    110     94    192     156    571    143    144   1633
3  F     110    134    132    186     145    549    131    135   1672
4  F      95     88     83    189     139    536    124    109   1700
5  M      83     94    100    180     163    549    141    124   1679
6  M     105     77     92    195     148    560    134    126   1651
>
> # Basic two-factor model
```



```
> mod1 = "Size =~ height + weight
+       Smart =~ verbal + progmat + reason"
>
```

\*This handout was prepared by Jerry Brunner, Department of Statistical Sciences, University of Toronto. It is licensed under a Creative Commons Attribution - ShareAlike 3.0 Unported License. Use any part of it as you like and share the result freely. The OpenOffice.org document is available from the course website:

<http://www.utstat.toronto.edu/brunner/oldclass/431s23>

```

> mod1 = "Size =~ height + weight
+       Smart =~ verbal + progmatt + reason"
>
> fit0 = cfa(mod1, data=bodymind); show(fit0)

```

lavaan 0.6.15 ended normally after 151 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	11
Number of observations	80

Model Test User Model:

Test statistic	1.753
Degrees of freedom	4
P-value (Chi-square)	0.781

```

>
> # Include sex as a grouping factor.
> # By default, the same model is fitted in all groups.
> # Different parameter estimates for M and F
> fit1 = cfa(mod1, data=bodymind, group = "sex")
> summary(fit1)

```

lavaan 0.6.15 ended normally after 406 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	32
Number of observations per group:	
M	37
F	43

Model Test User Model:

Test statistic	7.766
Degrees of freedom	8
P-value (Chi-square)	0.457
Test statistic for each group:	
M	4.106
F	3.661

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Group 1 [M]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
Size =~				
height	1.000			
weight	0.340	0.249	1.364	0.173
Smart =~				
verbal	1.000			
progmat	0.210	0.063	3.328	0.001
reason	0.552	0.135	4.099	0.000

Covariances:

	Estimate	Std.Err	z-value	P(> z )
Size ~~				
Smart	378.905	318.202	1.191	0.234

Intercepts:

	Estimate	Std.Err	z-value	P(> z )
.height	1692.027	12.366	136.830	0.000
.weight	132.216	4.014	32.937	0.000
.verbal	99.919	4.315	23.158	0.000
.progmat	101.432	1.384	73.306	0.000
.reason	97.595	2.603	37.491	0.000
Size	0.000			
Smart	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z )
.height	2521.668	2320.876	1.087	0.277
.weight	233.604	265.262	0.881	0.379
.verbal	149.488	115.352	1.296	0.195
.progmat	47.029	12.058	3.900	0.000
.reason	86.524	39.140	2.211	0.027
Size	3136.241	2535.627	1.237	0.216
Smart	539.290	191.140	2.821	0.005

Group 2 [F]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
Size =~				
height	1.000			
weight	0.228	0.102	2.234	0.026
Smart =~				
verbal	1.000			
progmat	0.382	0.077	4.977	0.000
reason	0.847	0.138	6.120	0.000

Covariances:

	Estimate	Std.Err	z-value	P(> z )
Size ~~				
Smart	460.680	205.132	2.246	0.025

Intercepts:

	Estimate	Std.Err	z-value	P(> z )
.height	1617.302	9.310	173.726	0.000
.weight	117.116	2.469	47.428	0.000
.verbal	99.372	3.667	27.099	0.000
.progmat	96.349	1.588	60.666	0.000
.reason	103.698	2.495	41.568	0.000
Size	0.000			
Smart	0.000			

Variances:

	Estimate	Std.Err	z-value	P(> z )
.height	1801.067	896.855	2.008	0.045
.weight	162.033	54.674	2.964	0.003
.verbal	218.547	61.771	3.538	0.000
.progmat	55.967	13.446	4.162	0.000
.reason	9.717	28.469	0.341	0.733
Size	1925.608	1071.706	1.797	0.072
Smart	359.686	122.164	2.944	0.003

```

>
> # Notice that intercepts are included. Model fit should not be affected.
> # Also notice Size ~~ Smart is significant for F but not M.
>
> # Test equal fit. Disregard intercepts and just set the following equal across groups:
>
> # lambda2, lambda4, lambda5, phi11, phi12, phi22, omega1, ..., omega5
> # There are eleven parameters, should be 11 df.
>
> # Parameters can be set equal by giving them the same names, but this is more convenient.
> fit2 = cfa(mod1, data=bodymind, group = "sex",
+           group.equal = c("loadings", "residuals", "lv.variances", "lv.covariances"))
> summary(fit2)

```

lavaan 0.6.15 ended normally after 166 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	32
Number of equality constraints	11
Number of observations per group:	
M	37
F	43

Model Test User Model:

Test statistic	22.327
Degrees of freedom	19
P-value (Chi-square)	0.268
Test statistic for each group:	
M	10.745
F	11.582

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Group 1 [M]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
Size =~				
height	1.000			
weight (.p2.)	0.242	0.097	2.505	0.012
Smart =~				
verbal	1.000			
progmat (.p4.)	0.303	0.051	5.961	0.000
reason (.p5.)	0.731	0.100	7.282	0.000

Covariances:

	Estimate	Std.Err	z-value	P(> z )
Size ~~				
Smart (.13.)	454.968	177.256	2.567	0.010

```
Intercepts:
      Estimate Std.Err z-value P(>|z|)
.height 1692.027  11.174 151.423  0.000
.weight  132.216   3.356  39.399  0.000
.verbal   99.919   4.124  24.227  0.000
.progmat 101.432   1.569  64.657  0.000
.reason   97.595   2.650  36.830  0.000
Size      0.000
Smart     0.000
```

```
Variances:
      Estimate Std.Err z-value P(>|z|)
.height (.p6.) 1609.534 1173.418  1.372  0.170
.weight (.p7.)  240.010  77.199  3.109  0.002
.verbal (.p8.)  216.159  55.713  3.880  0.000
.progmat (.p9.)   53.135   9.408  5.648  0.000
.reason (.10.)   39.040  24.142  1.617  0.106
Size    (.11.) 3010.342 1334.527  2.256  0.024
Smart   (.12.)  413.201 103.296  4.000  0.000
```

Group 2 [F]:

```
Latent Variables:
      Estimate Std.Err z-value P(>|z|)
Size =~
  height      1.000
  weight (.p2.) 0.242   0.097  2.505  0.012
Smart =~
  verbal      1.000
  progmat (.p4.) 0.303   0.051  5.961  0.000
  reason (.p5.) 0.731   0.100  7.282  0.000
```

```
Covariances:
      Estimate Std.Err z-value P(>|z|)
Size ~~
  Smart (.13.) 454.968 177.256  2.567  0.010
```

```
Intercepts:
      Estimate Std.Err z-value P(>|z|)
.height 1617.302  10.365 156.031  0.000
.weight  117.116   3.113  37.623  0.000
.verbal   99.372   3.826  25.975  0.000
.progmat  96.349   1.455  66.209  0.000
.reason  103.698   2.458  42.188  0.000
Size      0.000
Smart     0.000
```

```
Variances:
      Estimate Std.Err z-value P(>|z|)
.height (.p6.) 1609.534 1173.418  1.372  0.170
.weight (.p7.)  240.010  77.199  3.109  0.002
.verbal (.p8.)  216.159  55.713  3.880  0.000
.progmat (.p9.)   53.135   9.408  5.648  0.000
.reason (.10.)   39.040  24.142  1.617  0.106
Size    (.11.) 3010.342 1334.527  2.256  0.024
Smart   (.12.)  413.201 103.296  4.000  0.000
```

```
> anova(fit1,fit2)
```

Chi-Squared Difference Test

	Df	AIC	BIC	Chisq	Chisq diff	RMSEA	Df diff	Pr(>Chisq)
fit1	8	3526.6	3602.8	7.7663				
fit2	19	3519.1	3569.2	22.3270	14.561	0.089958	11	0.2035

```
> # Data are more likely when the model is less restricted, so fit1-fit2
> 2*(logLik(fit1) - logLik(fit2) )
'log Lik.' 14.56067 (df=32)
```

```

> # You might think the G^2 test of fit for fit0 and fit2 should be the same, but it can't
be. Fit0 does not know about the separate sample covariance matrices for M and F.
>
> # Test difference in just Size ~~ Smart
>
> mod2 = "Size =~ height + weight
+       Smart =~ verbal + progmatt + reason
+       Size ~~ c(phi12,phi12)*Smart" # Vector of parameter names
>
> fit3 = cfa(mod2, data=bodymind, group = "sex")
> summary(fit3)

```

lavaan 0.6.15 ended normally after 397 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	32
Number of equality constraints	1
Number of observations per group:	
M	37
F	43

Model Test User Model:

Test statistic	7.812
Degrees of freedom	9
P-value (Chi-square)	0.553
Test statistic for each group:	
M	4.139
F	3.673

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Group 1 [M]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
Size =~				
height	1.000			
weight	0.318	0.195	1.634	0.102
Smart =~				
verbal	1.000			
progmatt	0.210	0.062	3.376	0.001
reason	0.556	0.132	4.209	0.000

Covariances:

	Estimate	Std.Err	z-value	P(> z )
Size ~~				
Smart (phi12)	438.973	170.704	2.572	0.010

Intercepts:

	Estimate	Std.Err	z-value	P(> z )
.height	1692.027	12.448	135.929	0.000
.weight	132.216	4.029	32.812	0.000
.verbal	99.919	4.343	23.006	0.000
.progmatt	101.432	1.387	73.143	0.000
.reason	97.595	2.615	37.327	0.000
Size	0.000			
Smart	0.000			

```

Variances:
      Estimate Std.Err z-value P(>|z|)
.height 2320.080 2116.386  1.096  0.273
.weight 255.511  215.364  1.186  0.235
.verbal  153.926  113.493  1.356  0.175
.progmat  47.073   12.056  3.904  0.000
.reason  84.980   38.766  2.192  0.028
Size    3413.053 2240.445  1.523  0.128
Smart   544.006  184.404  2.950  0.003

```

Group 2 [F]:

```

Latent Variables:
      Estimate Std.Err z-value P(>|z|)
Size =~
  height      1.000
  weight      0.232    0.105    2.222    0.026
Smart =~
  verbal      1.000
  progmat     0.384    0.077    4.955    0.000
  reason      0.851    0.139    6.123    0.000

```

```

Covariances:
      Estimate Std.Err z-value P(>|z|)
Size ~~
Smart (ph12) 438.973 170.704  2.572  0.010

```

```

Intercepts:
      Estimate Std.Err z-value P(>|z|)
.height 1617.302  9.257 174.704  0.000
.weight  117.116  2.464  47.531  0.000
.verbal   99.372  3.647  27.251  0.000
.progmat   96.349  1.584  60.830  0.000
.reason  103.698  2.481  41.793  0.000
Size       0.000
Smart      0.000

```

```

Variances:
      Estimate Std.Err z-value P(>|z|)
.height 1826.478  887.640  2.058  0.040
.weight  160.697  55.193  2.912  0.004
.verbal  219.343  61.749  3.552  0.000
.progmat  56.040  13.465  4.162  0.000
.reason   9.246  28.622  0.323  0.747
Size    1858.603 1010.849  1.839  0.066
Smart   352.437  115.439  3.053  0.002

```

```
> anova(fit1,fit3)
```

Chi-Squared Difference Test

```

      Df    AIC    BIC  Chisq  Chisq diff  RMSEA  Df diff  Pr(>Chisq)
fit1   8 3526.6 3602.8 7.7663
fit3   9 3524.6 3598.5 7.8117  0.045402    0      1    0.8313

```

> # There are also tricks for setting a lot of parameters equal across groups, and then freeing some of them, allowing them to be different. A Google search for lavaan tutorial takes you to the right document.

```

> # Try radically different models for M and F, just as an experiment.
>
> Twomods =
+ "
+ group: 1
+ progmatt ~ verbal + theta*reason
+
+ group: 2
+ Size =~ height + weight
+ Smart =~ verbal + progmatt + theta*reason
+ "
>
> fit4 = cfa(Twomods, data=bodymind, group = "sex")
> summary(fit4)

```

lavaan 0.6.13 ended normally after 217 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	20
Number of equality constraints	1

Number of observations per group:	
M	37
F	43

Model Test User Model:

Test statistic	25.300
Degrees of freedom	5
P-value (Chi-square)	0.000
Test statistic for each group:	
M	13.896
F	11.404

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Group 1 [M]:

Regressions:

	Estimate	Std.Err	z-value	P(> z )
progmatt ~				
verbal	-0.057	0.062	-0.920	0.357
reason (thet)	0.525	0.074	7.048	0.000

Intercepts:

	Estimate	Std.Err	z-value	P(> z )
.progmatt	55.928	6.867	8.145	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.progmatt	73.081	16.991	4.301	0.000

Group 2 [F]:

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
Size =~				
height	1.000			
weight	0.236	0.118	2.005	0.045
Smart =~				
verbal	1.000			
progmatt	0.312	0.063	4.997	0.000
reason (thet)	0.525	0.074	7.048	0.000

Covariances:



	Estimate	Std.Err	z-value	P(> z )
Size ~				
Smart	518.447	244.497	2.120	0.034
Intercepts:				
	Estimate	Std.Err	z-value	P(> z )
.height	1617.302	9.310	173.726	0.000
.weight	117.116	2.469	47.428	0.000
.verbal	99.372	3.907	25.432	0.000
.progmatt	96.349	1.588	60.666	0.000
.reason	103.698	2.263	45.819	0.000
Size	0.000			
Smart	0.000			
Variances:				
	Estimate	Std.Err	z-value	P(> z )
.height	1863.466	971.613	1.918	0.055
.weight	158.679	59.888	2.650	0.008
.verbal	119.215	69.251	1.721	0.085
.progmatt	56.034	14.025	3.995	0.000
.reason	72.123	23.726	3.040	0.002
Size	1863.209	1125.596	1.655	0.098
Smart	537.289	149.070	3.604	0.000