Factor Combination Rule

Suppose the parameters of two factor analysis models (for disjoint sets of observable variables) are identifiable. Write the models as

$$d_1 = \Lambda_1 F_1 + e_1 \qquad d_3 = \Lambda_3 F_2 + e_3$$
$$d_2 = \Lambda_2 F_1 + e_2 \qquad d_4 = \Lambda_4 F_2 + e_4$$

where F_1 and d_1 are $p_1 \times 1$, and F_2 & d_3 are $p_2 \times 1$. It's natural for d_1 to contain the reference variables for F_1 & d_3 for F_2, but that's not necc. Just let Λ_1^{-1} and Λ_3^{-1} exist, & $cov(e_1, e_3) = 0$

	d_1	d_2	d_3	d_4
d_1	$\Lambda_1 \Phi_1 \Lambda_1^T + \Omega_{11}$	$\Lambda_1 \Phi_1 \Lambda_2^T + \Omega_{12}$	$\Lambda_1 \Phi_{12} \Lambda_3^T$	$\Lambda_1 \Phi_{12} \Lambda_4 + \Omega_{14}$
d_2		$\Lambda_2 \Phi_{11} \Lambda_2^T + \Omega_{22}$	$+ \Omega_{23}$	$+ \Omega_{24}$
d_3			$\Lambda_3 \Phi_2 \Lambda_3^T + \Omega_{33}$	$\Lambda_3 \Phi_2 \Lambda_4^T + \Omega_{34}$
d_4				$\Lambda_4 \Phi \Lambda_4^T + \Omega_{44}$

Got it, but what about cross-over

Extra variables Rule
(enhanced crossover)

Have $d_0 = \Lambda_0 F + e_1$ ident.

Re-write as $d_1 = \Lambda_1 F + e_1$ pxp, inverse exists

$d_2 = \Lambda_2 F + e_2$

Add $d_3 = \Lambda_3 F + e_3$, $cov(e_1, e_3) = 0$

$\Sigma_{13} = cov(d_1 d_3) = \Lambda_1 \Phi \Lambda_3^T$ get Λ_3 iden

$\Sigma_{23} = cov(d_2 d_3) = \Lambda_2 \Phi \Lambda_3^T + \Omega_{23}$ ok!

ident.

Vector 3-var Rule

ref vars & re-para

$$d_1 = F + e_1$$

$$d_2 = \Lambda_2 F + e_2$$

$$d_3 = \Lambda_3 F + e_3$$

all $p \times 1$

$\mathrm{cov}(F, e_i) = \mathrm{cov}(e_i, e_j) = 0$

$\Lambda_2\ \Lambda_3$ have inverses

	d_1	d_2	d_3
d_1		$\Phi \Lambda_2^T$	$\Phi \Lambda_3^T$
d_2			$\Lambda_2 \Phi \Lambda_3^T$
d_3			

$$\Phi\ \Sigma_{12} \Sigma_{13} \Sigma_{23}^{-1}$$

This is better than it looks

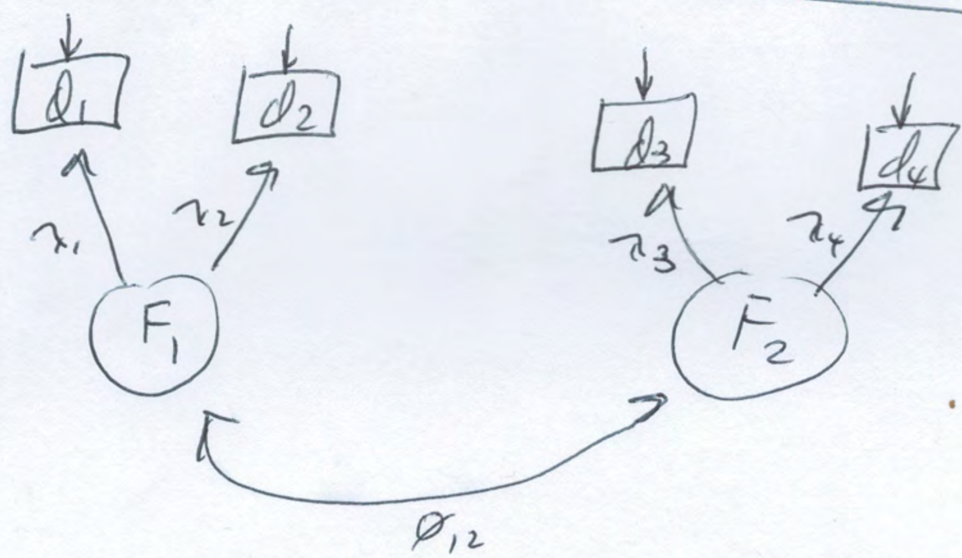
Make Λ_2 diagonal

$$= \Phi \Lambda_2^T \Phi \Lambda_3^T (\Lambda_2 \Phi \Lambda_3^T)^{-1}$$

$$= \Phi \Lambda_2^T \Phi \Lambda_3^T \Lambda_3^{T^{-1}} \Phi^{-1} \Lambda_2^{-1}$$

$$= \Phi \Lambda_2^T \Lambda_2^{-1}$$

Two-variable Rule for Standardised Factors



$d_1 = \lambda_1 F_1 + e_1$
$d_2 = \lambda_2 F_1 + e_2$
$d_3 = \lambda_3 F_2 + e_3$
$d_4 = \lambda_4 F_2 + e_4$

$Var(F_1) = Var(F_2) = 1$

$\lambda_1 > 0, \ \lambda_3 > 0$

	d_1	d_2	d_3	d_4
d_1	$\lambda_1^2 + \omega_1$	$\lambda_1 \lambda_2$	$\lambda_1 \lambda_3 \phi_{12}$	$\lambda_1 \lambda_4 \phi_{12}$
d_2		$\lambda_2^2 + \omega_2$	$\lambda_2 \lambda_3 \phi_{12}$	$\lambda_2 \lambda_4 \phi_{12}$
d_3			$\lambda_3^2 + \omega_3$	$\lambda_3 \lambda_4$
d_4				$\lambda_4^2 + \omega_4$

$\dfrac{\sigma_{14}\sigma_{23}}{\sigma_{12}\sigma_{34}} = \phi_{12}^2$, recover sign from σ_{13}, get ϕ_{12}

$\dfrac{\sigma_{13}\sigma_{14}}{\sigma_{34}} = \dfrac{\lambda_1^2 \lambda_3 \lambda_4 \phi_{12}^2}{\lambda_3 \lambda_4} = \lambda_1^2 \phi_{12}^2$ Divide by ϕ_{12}^2, get λ_1^2

But $\lambda_1 > 0$ so have λ_1

Get λ_2 from σ_{12}

$\sigma_{13}\sigma_{23} = \lambda_1 \lambda_2 \phi_{12}^2 \lambda_3^2$ but $\lambda_3 > 0$ so get λ_3, get λ_4 from σ_{34}