STA 431 Assignment 10

① (a)

$$\Sigma = $$

	D_1	D_2	D_3
D_1	$\lambda_1^2 + \omega_1$	0	0
D_2	0	$\lambda_2^2 + \omega_2$	0
D_3	0	0	$\lambda_3^2 + \omega_3$

(b) No. There are 3 equations in 6 unknown parameters. The model fails the parameter count rule.

(c) Test $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$

② (a)

$$\Sigma = $$

	D_1	D_2	D_3
D_1	$\lambda_1^2 + \omega_1$	$\lambda_1 \lambda_2$	$\lambda_1 \lambda_3$
D_2		$\lambda_2^2 + \omega_2$	$\lambda_2 \lambda_3$
D_3			$\lambda_3^2 + \omega_3$

(b) Yes: $\frac{\sigma_{12}\sigma_{13}}{\sigma_{23}} = \lambda_1^2 \Rightarrow \lambda_1 = \sqrt{\frac{\sigma_{12}\sigma_{13}}{\sigma_{23}}}$ Because $\lambda_1 > 0$

$\lambda_2 = \sigma_{12} / \lambda_1$

$\lambda_3 = \sigma_{13} / \lambda_1$

Get ω_1, ω_2 and ω_3 by subtraction

(3) Now $\Sigma =$

	D_1	D_2	D_3	D_4
D_1	$\lambda_1^2+\omega_1$	$\lambda_1\lambda_2=0$	$\lambda_1\lambda_3$	$\lambda_1\lambda_4$
D_2		$\lambda_2^2+\omega_2$	$\lambda_2\lambda_3=0$	$\lambda_2\lambda_4=0$
D_3			$\lambda_3^2+\omega_3$	$\lambda_3\lambda_4$
D_4				$\lambda_4^2+\omega_4$

(a) Yes. Because other factor loadings are non-zero, $\lambda_2 = 0$ iff $\sigma_{12} = \sigma_{23} = \sigma_{24} = 0$.

(b) Yes. The other parameters may be identified as in Problem 2.

(4) Now $\Sigma =$

	D_1	D_2	D_3	D_4	D_5
D_1	$\lambda_1^2+\omega_1$	$\lambda_1\lambda_2$	0	0	$\lambda_1\lambda_5$
D_2		$\lambda_2^2+\omega_2$	0	0	$\lambda_2\lambda_5$
D_3			$\lambda_3^2+\omega_3$	0	0
D_4				$\lambda_4^2+\omega_4$	0
D_5					$\lambda_5^2+\omega_5$

(a) Yes. Because $\lambda_1 > 0$, $\lambda_3 = 0$ iff $\sigma_{13} = cov(D_1 D_3) = 0$ and $\lambda_4 = 0$ iff $\sigma_{14} = 0$

(b) Yes. λ_1, λ_2 & λ_5 may be identified as in Problem 2.

(c) All parameters are identifiable provided at least 3 factor loadings are non-zero and the sign of one factor loading is known.

(5)

(a)

$$\Sigma =$$

	D_1	D_2	D_3	D_4	D_5	D_6
D_1	$\lambda_1^2+w_1$	$\lambda_1\lambda_2$	$\lambda_1\lambda_3$	$\lambda_1\lambda_4\phi_{12}$	$\lambda_1\lambda_5\phi_{12}$	$\lambda_1\lambda_6\phi_{12}$
D_2		$\lambda_2^2+w_2$	$\lambda_2\lambda_3$	$\lambda_2\lambda_4\phi_{12}$	$\lambda_2\lambda_5\phi_{12}$	$\lambda_2\lambda_6\phi_{12}$
D_3			$\lambda_3^2+w_3$	$\lambda_3\lambda_4\phi_{12}$	$\lambda_3\lambda_5\phi_{12}$	$\lambda_3\lambda_6\phi_{12}$
D_4				$\lambda_4^2+w_4$	$\lambda_4\lambda_5$	$\lambda_4\lambda_6$
D_5					$\lambda_5^2+w_5$	$\lambda_5\lambda_6$
D_6						$\lambda_6^2+w_6$

$$\sigma_{14} = \text{Cov}(D_1, D_4) = \text{cov}(\lambda_1 F_1 + e_1, \lambda_4 F_2 + e_2)$$
$$= \lambda_1 \lambda_4 \phi_{12}$$

(b) No. $\{\lambda_j\}$ and $\{-\lambda_j\}$ yield the same Σ.

(c)
$$\begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ D_6 \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ \lambda_2 & 0 \\ \lambda_3 & 0 \\ 0 & \lambda_4 \\ 0 & \lambda_5 \\ 0 & \lambda_6 \end{pmatrix} \begin{pmatrix} F_1 \\ F_2 \end{pmatrix} + \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{pmatrix}$$

$$d = \Lambda F + e$$

(5d)

$$\begin{pmatrix} \lambda_1 & 0 \\ \lambda_2 & 0 \\ \lambda_3 & 0 \\ 0 & \lambda_4 \\ 0 & \lambda_5 \\ 0 & \lambda_6 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -\lambda_1 & 0 \\ -\lambda_2 & 0 \\ -\lambda_3 & 0 \\ 0 & -\lambda_4 \\ 0 & -\lambda_5 \\ 0 & -\lambda_6 \end{pmatrix}$$

$$\Lambda \qquad R \qquad = \qquad \Lambda_2 = -\Lambda$$

Note

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$R \qquad R^T \qquad = \qquad I$$

(e) Yes. Identify $\lambda_1, \lambda_2, \lambda_3$ from $cov\begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}$ and identify $\lambda_4, \lambda_5, \lambda_6$ from $cov\begin{pmatrix} D_4 \\ D_5 \\ D_6 \end{pmatrix}$, then identify ϕ_{12} from σ_{14} or any other unused covariance.

(f) $6(6+1)/2 - 13 = 21 - 13 = 8$ df

(6) Yes, provided $\phi_{12} \neq 0$. Two-variable addition rule.

(7) Yes. λ_7, λ_8 & λ_9 are identified from $cov\begin{pmatrix} D_7 \\ D_8 \\ D_9 \end{pmatrix}$, and ϕ_{13} & ϕ_{23} are identified from, say σ_{17} and σ_{47}.

(8)

(a) Σ =

	D_1	D_2	D_3
D_1	$\phi + w_1$	$\lambda_2 \phi$	$\lambda_3 \phi$
D_2		$\lambda_2^2 \phi + w_2$	$\lambda_2 \lambda_3 \phi$
D_3			$\lambda_3^2 \phi + w_3$

b) Yes $\phi = \frac{\sigma_{12}\sigma_{13}}{\sigma_{23}}$, $\lambda_2 = \frac{\sigma_{12}}{\phi}$, $\lambda_3 = \frac{\sigma_{13}}{\phi}$

w_1, w_2, w_3 by subtraction.

(9) (a) Σ =

	D_1	D_2	D_3	D_4	D_5	D_6
D_1	$\phi_{11} + w_1$	$\lambda_2 \phi_{11}$	$\lambda_3 \phi_{11}$	ϕ_{12}	$\lambda_5 \phi_{12}$	$\lambda_6 \phi_{12}$
D_2		$\lambda_2^2 \phi_{11} + w_2$	$\lambda_2 \lambda_3 \phi_{11}$	$\lambda_2 \phi_{12}$	$\lambda_2 \lambda_5 \phi_{12}$	$\lambda_2 \lambda_6 \phi_{12}$
D_3			$\lambda_3^2 \phi_{11} + w_3$	$\lambda_5 \phi_{12}$	$\lambda_3 \lambda_5 \phi_{12}$	$\lambda_3 \lambda_6 \phi_{12}$
D_4				$\phi_{22} + w_4$	$\lambda_4 \phi_{22}$	$\lambda_5 \phi_{22}$
D_5					$\lambda_5^2 \phi_{22} + w_5$	$\lambda_4 \lambda_5 \phi_{22}$
D_6						$\lambda_6^2 \phi_{22} + w_6$

(9b)

$\phi_{11}, \omega_1, \omega_2, \omega_3, \lambda_2$ & λ_3 are identified from $cov\begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix}$ as in Problem 8.

$\phi_{22}, \omega_4, \omega_5, \omega_6, \lambda_5$ & λ_6 are identified from $cov\begin{pmatrix} D_4 \\ D_5 \\ D_6 \end{pmatrix}$ as in Problem 8.

The only remaining parameter is $\phi_{12} = \sigma_{14}$.

(10) Yes! I almost can't stand this any more.

(11) (a.)

$$d_1 = \gamma_1 + \lambda_1 F_1 + e_1$$
$$d_2 = \gamma_2 + \lambda_2 F_1 + e_2$$
$$d_3 = \gamma_3 + \lambda_3 F_2 + e_3$$
$$d_4 = \gamma_4 + \lambda_4 F_2 + e_4$$

(b)

$$d_1 = \gamma_1 + (\lambda_1 F_1) + e_1 = \gamma_1 + F_1' + e_1$$
$$d_3 = \gamma_3 + (\lambda_3 F_2) + e_3 = \gamma_3 + F_2' + e_3$$
$$\phi_{12}' = cov(F_1', F_2') = cov(\lambda_1 F_1, \lambda_3 F_2)$$
$$= \lambda_1 \lambda_3 \phi_{12}$$

(11c) $d_1 = F_1 + e_1$ $\quad d_3 = F_2 + e_3$

$d_2 = \lambda_2 F_1 + e_2$ $\quad d_4 = \lambda_4 F_2 + e_4$

$\Sigma =$

	d_1	d_2	d_3	d_4
d_1	$\phi_{11} + w_1$	$\lambda_2 \phi_{11}$	ϕ_{12}	$\lambda_4 \phi_{12}$
d_2		$\lambda_2^2 \phi_{11} + w_2$	$\lambda_2 \phi_{12}$	$\lambda_2 \lambda_4 \phi_{12}$
d_3			$\phi_{22} + w_3$	$\lambda_4 \phi_{22}$
d_4				$\lambda_4^2 \phi_{22} + w_4$

(d) $\phi_{12} = \sigma_{13}$, $\lambda_4 = \sigma_{14}/\phi_{12}$, $\lambda_2 = \sigma_{23}/\phi_{12}$

$\phi_{11} = \sigma_{12}/\lambda_2$, $\phi_{22} = \sigma_{34}/\lambda_4$

$w_1 = \sigma_{11} - \phi_{11}$, similar for w_2, w_3 & w_4

(e) $4(4+1)/2 - 9 = 1$

(f) $\sigma_{24} = \lambda_2 \lambda_4 \phi_{12} = \lambda_2 \sigma_{14} = \frac{\sigma_{23}}{\sigma_{13}} \sigma_{14}$

$\Longrightarrow \sigma_{13}\sigma_{24} = \sigma_{23}\sigma_{14}$

(g) $(\lambda_4 \phi_{12})(\lambda_2 \lambda_4 \phi_{12}) = (\lambda_2 \phi_{12})(\lambda_4 \phi_{12})$

If $\lambda_2 = 0$, both sides are zero

If $\lambda_4 = 0$, " " "

If $\phi_{12} = 0$, " " " "

12

(a)

$\Sigma =$

	d_1	d_2	d_3
d_1	$\Lambda_1 \Phi \Lambda_1^T + \Omega_{11}$	$\Lambda_1 \Phi \Lambda_2^T$	$\Lambda_1 \Phi \Lambda_3^T$
d_2		$\Lambda_2 \Phi \Lambda_2^T + \Omega_{22}$	$\Lambda_2 \Phi \Lambda_3^T + \Omega_{23}$
d_3			$\Lambda_3 \Phi \Lambda_3^T + \Omega_{33}$

(b) $\Lambda_3^T = (\Lambda_1 \Phi)^{-1} \Sigma_{13}$, so $\Lambda_3 = \Sigma_{31} (\Phi^{-1} \Lambda_1^{-1})^T$

$= \Sigma_{31} \Lambda_1^{-1T} \Phi^{-1}$

$\Omega_{33} = \Sigma_{33} - \Lambda_3 \Phi \Lambda_3^T$

$\Omega_{23} = \Sigma_{23} - \Lambda_2 \Phi \Lambda_3^T$

(13)

$$\Sigma = $$

	d_1	d_2	d_3	d_4
d_1	$\Lambda_1 \Phi_{11} \Lambda_1^T + \Omega_{11}$	$\Lambda_1 \Phi_{11} \Lambda_2^T + \Omega_{23}$	$\Lambda_1 \Phi_{12} \Lambda_3^T$	$\Lambda_1 \Phi_{12} \Lambda_4^T + \Omega_{14}$
d_2		$\Lambda_2 \Phi_{11} \Lambda_2^T + \Omega_{22}$	$\Lambda_2 \Phi_{12} \Lambda_3^T + \Omega_{23}$	$\Lambda_2 \Phi \Lambda_4^T + \Omega_{24}$
d_3			$\Lambda_3 \Phi_{22} \Lambda_3^T + \Omega_{33}$	$\Lambda_3 \Phi_{22} \Lambda_4^T + \Omega_{44}$
d_4				$\Lambda_4 \Phi_{22} \Lambda_3^T + \Omega_{33}$

(a) Φ_{12}, Ω_{14}, Ω_{23}

(b) $\Phi_{12} = \Lambda_1^{-1} \Sigma_{13} (\Lambda_3^T)^{-1}$

(c) $\Omega_{14} = \Sigma_{14} - \Lambda_1 \Phi_{12} \Lambda_4^T$

(d) $\Omega_{23} = \Sigma_{23} - \Lambda_2 \Phi_{12} \Lambda_3^T$

(e) $\Omega_{24} = \Sigma_{24} - \Lambda_2 \Phi_{12} \Lambda_4^T$

(14) (a)

$D_1 = \gamma_1 + \lambda_1 F_1 + e_1$ $\quad D_4 = \gamma_4 + \lambda_4 F_2 + e_4$

$D_2 = \gamma_2 + \lambda_2 F_1 + e_2$ $\quad D_5 = \gamma_5 = \lambda_5 F_2 + e_5$

$D_3 = \gamma_3 + \lambda_3 F_1 + e_3$ $\quad D_6 = \gamma_6 + \lambda_6 F_2 + e_6$

(b) i) $D_1 = \lambda_1 F_1 + e_1$

$$= \lambda_1 \sqrt{\phi_{11}} \left(\frac{1}{\sqrt{\phi_{11}}} F_1 \right) + e_1$$

$$= \left(\lambda_1 \sqrt{\phi_{11}} \right) F_1' + e_1 \quad \text{so } var(F_1') = 1$$

$$D_2 = \left(\lambda_2 \sqrt{\phi_{11}} \right) F_1' + e_2$$

$$D_3 = \left(\lambda_3 \sqrt{\phi_{11}} \right) F_1' + e_3$$

$$D_4 = \left(\lambda_4 \sqrt{\phi_{22}} \right) F_2' + e_4$$

$$D_5 = \left(\lambda_5 \sqrt{\phi_{22}} \right) F_2' + e_5$$

$$D_6 = \left(\lambda_6 \sqrt{\phi_{22}} \right) F_2' + e_6$$

ii) $\phi_{12}' = Cov(F_1', F_2') = cov\left(\frac{1}{\sqrt{\phi_{11}}} F_1, \frac{1}{\sqrt{\phi_{22}}} F_2 \right)$

$$= \frac{1}{\sqrt{\phi_{11}}} \frac{1}{\sqrt{\phi_{22}}} Cov(F_1, F_2)$$

$$= \frac{\phi_{12}}{\sqrt{\phi_{11}} \sqrt{\phi_{22}}} = Corr(F_1, F_2)$$

iii) The ω_j are not affected.

(14c) $Var(D_2) = Var(\nu_2 + \lambda_2 F_2 + e_2)$

$$= \lambda_2^2 \phi_{11} + \omega_2$$

So explained variance is $\lambda_2^2 \phi_{11}$

(d) $Var(D_2) = Var(\lambda_2' F_2' + e_2)$

$$= (\lambda_2')^2 + \omega_2$$

So explained variance is $(\lambda_2')^2$

(e) $(\lambda_2')^2 = (\sqrt{\phi_{11}}\, \lambda_2)^2 = \lambda_2^2 \phi_{11}$,

So No. Amount of variance explained is not affected.

```
R version 4.2.3 (2023-03-15) -- "Shortstop Beagle"
Copyright (C) 2023 The R Foundation for Statistical Computing
Platform: x86_64-apple-darwin17.0 (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.79 (8198) x86_64-apple-darwin17.0]

[Workspace restored from /Users/brunner/.RData]
[History restored from /Users/brunner/.Rapp.history]

> # CFA of poverty data
> # install.packages("lavaan", dependencies = TRUE) # Only need to do this once
> library(lavaan)
This is lavaan 0.6-15
lavaan is FREE software! Please report any bugs.
> 
> rm(list=ls())
> # pov = read.table("poverty.data.txt") # Local copy
> pov = read.table("https://www.utstat.toronto.edu/brunner/data/illegal/poverty.data.txt",
na.strings = ".")
> 
> head(pov)
    V1   V2   V3   V4   V5   V6 V7                V8
1 24.7  5.7 30.8 69.6 75.5  600  1           Albania
2 12.5 11.9 14.4 68.3 74.7 2250  1          Bulgaria
3 13.4 11.7 11.3 71.8 77.7 2980  1    Czechoslovakia
4 12.0 12.4  7.6 69.8 75.9   NA  1 Former_E._Germany
5 11.6 13.4 14.8 65.4 73.8 2780  1           Hungary
6 14.3 10.2 16.0 67.2 75.7 1690  1            Poland
> colnames(pov) = c("birthrate", "deathrate", "infmort", "lifexM", "lifexF", "gnp",
"continent", "country")
> 
> # Data transformations
> pov = within(pov,{
+ LifeEx = (lifexM+lifexF)/2
+ gnp1000 = gnp/1000
+ }) # End within pov
> 
> summary(pov)
   birthrate       deathrate        infmort          lifexM          lifexF
 Min.   : 9.70   Min.   : 2.20   Min.   :  4.5   Min.   :38.10   Min.   :41.20
 1st Qu.:14.50   1st Qu.: 7.80   1st Qu.: 13.1   1st Qu.:55.80   1st Qu.:57.50
```

```
 Median :29.00   Median : 9.50   Median : 43.0   Median :63.70   Median :67.80  
 Mean   :29.23   Mean   :10.84   Mean   : 54.9   Mean   :61.49   Mean   :66.15  
 3rd Qu.:42.20   3rd Qu.:12.50   3rd Qu.: 83.0   3rd Qu.:68.60   3rd Qu.:75.40  
 Max.   :52.20   Max.   :25.00   Max.   :181.6   Max.   :75.90   Max.   :81.80  
                                                                                
      gnp          continent       country              gnp1000          LifeEx     
 Min.   :   80   Min.   :1.000   Length:97          Min.   : 0.080   Min.   :39.65  
 1st Qu.:  475   1st Qu.:3.000   Class :character   1st Qu.: 0.475   1st Qu.:56.25  
 Median : 1690   Median :4.000   Mode  :character   Median : 1.690   Median :65.80  
 Mean   : 5741   Mean   :3.948                      Mean   : 5.741   Mean   :63.82  
 3rd Qu.: 7325   3rd Qu.:6.000                      3rd Qu.: 7.325   3rd Qu.:71.55  
 Max.   :34064   Max.   :6.000                      Max.   :34.064   Max.   :78.85  
 NA's   :6                                          NA's   :6                       
> 
> # Use LifeEx, infmort, birthrate, gnp1000
> 
> mod = "Health =~ LifeEx + infmort
+        Wealth =~ gnp1000 + birthrate "
> 
> fit1 = cfa(mod,std.lv=TRUE, data=pov) # Standardizing latent variables
> summary(fit1)
lavaan 0.6.15 ended normally after 47 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         9

                                                  Used       Total
  Number of observations                            91          97

Model Test User Model:
                                                      
  Test statistic                                 1.098
  Degrees of freedom                                 1
  P-value (Chi-square)                           0.295

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)
  Health =~                                           
    LifeEx           10.256    0.775   13.225    0.000
    infmort         -44.025    3.577  -12.309    0.000
  Wealth =~                                           
    gnp1000           5.454    0.758    7.196    0.000
    birthrate       -12.648    1.140  -11.091    0.000

Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)
  Health ~~                                           
    Wealth            0.961    0.030   31.660    0.000
```

```
Variances:
                   Estimate  Std.Err  z-value  P(>|z|)
   .LifeEx            1.432    1.815    0.789    0.430
   .infmort         182.139   42.803    4.255    0.000
   .gnp1000          35.041    5.484    6.389    0.000
   .birthrate        25.637   10.190    2.516    0.012
    Health            1.000
    Wealth            1.000

> # Fairly close to proc calis values from 2013, very close to 2017
> # Ah I was using vardef=n in 2017.
> parameterEstimates(fit1)
         lhs op       rhs     est     se       z pvalue ci.lower ci.upper
1     Health =~    LifeEx  10.256  0.775  13.225  0.000    8.736   11.775
2     Health =~   infmort -44.025  3.577 -12.309  0.000  -51.035  -37.015
3     Wealth =~   gnp1000   5.454  0.758   7.196  0.000    3.968    6.940
4     Wealth =~ birthrate -12.648  1.140 -11.091  0.000  -14.883  -10.413
5     LifeEx ~~    LifeEx   1.432  1.815   0.789  0.430   -2.125    4.988
6    infmort ~~   infmort 182.139 42.803   4.255  0.000   98.247  266.032
7    gnp1000 ~~   gnp1000  35.041  5.484   6.389  0.000   24.292   45.791
8  birthrate ~~ birthrate  25.637 10.190   2.516  0.012    5.665   45.610
9     Health ~~    Health   1.000  0.000      NA     NA    1.000    1.000
10    Wealth ~~    Wealth   1.000  0.000      NA     NA    1.000    1.000
11    Health ~~    Wealth   0.961  0.030  31.660  0.000    0.901    1.020
>
> fit2 = cfa(mod, data=pov) # Setting first factor loading to one (default)
> summary(fit2, standardized=TRUE)
lavaan 0.6.15 ended normally after 109 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         9

                                                  Used       Total
  Number of observations                            91          97

Model Test User Model:

  Test statistic                                 1.098
  Degrees of freedom                                 1
  P-value (Chi-square)                           0.295

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Health =~
    LifeEx            1.000                               10.256    0.993
    infmort          -4.293    0.164  -26.155    0.000  -44.025   -0.956
  Wealth =~
    gnp1000           1.000                                5.454    0.678
    birthrate        -2.319    0.294   -7.875    0.000  -12.648   -0.928
```

```
Covariances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Health ~~
    Wealth           53.733   10.371    5.181    0.000    0.961    0.961

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .LifeEx            1.432    1.815    0.789    0.430    1.432    0.013
   .infmort         182.139   42.803    4.255    0.000  182.139    0.086
   .gnp1000          35.041    5.484    6.389    0.000   35.041    0.541
   .birthrate        25.637   10.190    2.516    0.012   25.637    0.138
    Health          105.176   15.906    6.613    0.000    1.000    1.000
    Wealth           29.746    8.268    3.598    0.000    1.000    1.000

> # Compare Sigma(thetahat) matrices. Illustrated this in lecture.
> fitted(fit1)
$cov
            LifeEx   infmrt   gn1000   brthrt
LifeEx     106.608
infmort   -451.501 2120.346
gnp1000     53.733 -230.666   64.788
birthrate -124.607  534.911  -68.981  185.604

> fitted(fit2)
$cov
            LifeEx   infmrt   gn1000   brthrt
LifeEx     106.608
infmort   -451.501 2120.346
gnp1000     53.733 -230.666   64.788
birthrate -124.607  534.911  -68.981  185.604

>
> # lambda2/lambda1 (sqrt{phi11} cancels)
> -44.025/10.256
[1] -4.292609
>
> # 15e (v) Proportion of variance in infant mortality rate explained by Health
> (-0.956)^2
[1] 0.913936
>
> ###### Single-factor model
> singlemod = "Wealth =~ gnp1000 + birthrate + LifeEx + infmort"
> fit3 = cfa(singlemod, data=pov)
> summary(fit3, standardized=TRUE)
lavaan 0.6.15 ended normally after 66 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                         8

                                                  Used       Total
  Number of observations                            91          97

Model Test User Model:
```

```
  Test statistic                                 2.823
  Degrees of freedom                                 2
  P-value (Chi-square)                           0.244

Parameter Estimates:

  Standard errors                             Standard
  Information                                 Expected
  Information saturated (h1) model          Structured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  Wealth =~
    gnp1000           1.000                               5.262    0.654
    birthrate        -2.313    0.308   -7.509    0.000  -12.172   -0.893
    LifeEx            1.947    0.241    8.092    0.000   10.243    0.992
    infmort          -8.374    1.058   -7.917    0.000  -44.062   -0.957

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .gnp1000          37.103    5.561    6.672    0.000   37.103    0.573
   .birthrate        37.453    6.083    6.157    0.000   37.453    0.202
   .LifeEx            1.679    1.709    0.982    0.326    1.679    0.016
   .infmort         178.867   41.144    4.347    0.000  178.867    0.084
    Wealth           27.685    7.916    3.497    0.000    1.000    1.000

>
> # With factanal, get same factor loadings (reflected), but a slightly differfent fit test.
Probably using Lawley's original test instead of G^2.
> dat = pov[c(1,3,9,10)]; colnames(dat)
[1] "birthrate" "infmort"   "gnp1000"   "LifeEx"
> factanal(na.omit(dat),factors=1)

Call:
factanal(x = na.omit(dat), factors = 1)

Uniquenesses:
birthrate   infmort   gnp1000    LifeEx
    0.202     0.084     0.573     0.016

Loadings:
          Factor1
birthrate  0.893
infmort    0.957
gnp1000   -0.654
LifeEx    -0.992

               Factor1
SS loadings      3.125
Proportion Var   0.781

Test of the hypothesis that 1 factor is sufficient.
The chi square statistic is 2.7 on 2 degrees of freedom.
The p-value is 0.259
>
>
```