Name Jerry

Student Number

STA 431S 2017 Quiz 8

1. (6 points) This is the matrix version of instrumental variables. Independently for $i = 1, \ldots, n$, the centered model equations are

$$
\begin{aligned}
\mathbf{Y}_i &= \beta\mathbf{X}_i + \epsilon_i \\
\mathbf{W}_i &= \mathbf{X}_i + \mathbf{e}_{i,1} \\
\mathbf{V}_i &= \mathbf{Y}_i + \mathbf{e}_{i,2}.
\end{aligned}
$$

The random vectors $\mathbf{X}_i$ and $\mathbf{Y}_i$ are latent, while $\mathbf{W}_i$ and $\mathbf{V}_i$ are observable. In addition, there is a vector of observable instrumental variables $\mathbf{Z}_i$. The random vectors $\mathbf{X}_i$ and $\mathbf{Z}_i$ are $p \times 1$, while $\mathbf{Y}_i$ is $q \times 1$. The variances and covariances are as follows: $cov(\mathbf{X}_i) = \mathbf{\Phi}_x$, $cov(\mathbf{Z}_i) = \mathbf{\Phi}_z$, $cov(\mathbf{Z}_i, \mathbf{X}_i) = \mathbf{\Phi}_{zx}$, $cov(\epsilon_i) = \mathbf{\Psi}$, $cov(\mathbf{X}_i, \epsilon_i) = \mathbf{C}$, and $cov\left(\begin{array}{c} \mathbf{e}_{i,1} \\ \mathbf{e}_{i,2} \end{array}\right) = \mathbf{\Omega}$. All variance-covariance matrices are positive definite, and in addition, the $p \times p$ matrix of covariances $\mathbf{\Phi}_{zx}$ has an inverse. Covariances that are not specified are zero; in particular, the instrumental variables have zero covariance with the error terms.

Collecting $\mathbf{Z}_i$, $\mathbf{W}_i$, $\mathbf{V}_i$ into a single long data vector $\mathbf{D}_i$,

$$
cov(\mathbf{D}_i) = \mathbf{\Sigma} = \begin{array}{c|c} & \begin{array}{ccc} Z & W & V \end{array} \\ \begin{array}{c} Z \\ W \\ V \end{array} & \left(\begin{array}{c|c|c} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} & \mathbf{\Sigma}_{13} \\ \hline & \mathbf{\Sigma}_{22} & \mathbf{\Sigma}_{23} \\ \hline & & \mathbf{\Sigma}_{33} \end{array}\right) \end{array},
$$

where $cov(\mathbf{Z}_i, \mathbf{W}_i) = \mathbf{\Sigma}_{12}$, and so on.

Prove that β is identifiable by showing how it can be recovered from the $\mathbf{\Sigma}_{ij}$ matrices.

There is more room on the reverse side if you need it.

$$cov(Z_i, V_i) = E\{Z_i(\beta X_i + \varepsilon_i + e_{i2})^T\}$$

$$= E\{Z_i(X_i^T\beta^T + \varepsilon_i^T + e_{i2}^T)\}$$

$$= E\{Z_iX_i^T\}\beta^T + E\{Z_i\}E\{\varepsilon_i^T\} + E\{Z_i\}E\{e_{i2}^T\}$$

$$= \Phi_{zx}\beta^T = \Sigma_{13} \text{ and}$$

$$cov(Z_i W_i) = E\{Z_i(X_i + e_{i1})^T\} = E\{Z_iX_i^T\} + 0 = \Sigma_{12}, \text{ so}$$

$$\beta = (\Sigma_{12}^{-1}\Sigma_{13})^T = \Sigma_{13}^T\Sigma_{12}^{-1T}$$

~~There is additional room here in case you need it, but you should not need it.~~

2. (4 points) In the SAS part of the assignment, you analyzed the Longitudinal IQ data, and tested equality of the regression coefficients linking mother's true academic ability to the child's IQ at different ages. Write the likelihood ratio test statistic G^2 in the space below.

$G^2 = 15.9033$

Please attach *both* your log file and your results file. Make sure your name appears on both documents. Attach the *log file*, not just a listing of the SAS program.