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Student Number

STA 431 s2017 Quiz 1

1. (5 points) Let $\mathbf{Y}$ be a $p \times 1$ random vector with expected value $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, and let $\mathbf{a}$ be a $p \times 1$ vector of constants. Choose *one* of the statements below and prove it. Start with the definition of a covariance matrix on the formula sheet. Do not use the Centering Rule on this question.

   (a) $cov(\mathbf{Y}+\mathbf{a}) = \boldsymbol{\Sigma}$

   (b) $cov(\mathbf{Y}+\mathbf{a}) = \boldsymbol{\Sigma}+\mathbf{a}$

   (c) $cov(\mathbf{Y}+\mathbf{a}) = \mathbf{0}$

   (d) $cov(\mathbf{Y}+\mathbf{a}) = \mathbf{a}\boldsymbol{\Sigma}\mathbf{a}^\top$

   (e) $cov(\mathbf{Y}+\mathbf{a}) = \mathbf{a}^\top\boldsymbol{\Sigma}\mathbf{a}$.

$E(Y+a) = \mu + a$, so

$$cov(Y+a) = E\{(Y+a-(\mu+a))(Y+a-(\mu+a))^T\}$$
$$= E\{(Y+\cancel{a}-\mu-\cancel{a})(Y+\cancel{a}-\mu-\cancel{a})^T\}$$
$$= E\{(Y-\mu)(Y-\mu)^T\} = cov(Y) = \Sigma$$

2. (5 points) Consider the scalar regression equation $Y = \beta_0 + \beta_1 X + \epsilon$, where $E(X) = \mu$, $Var(X) = \sigma_x^2$, $E(\epsilon) = 0$, $Var(\epsilon) = \sigma_\epsilon^2$, and $Cov(X, \epsilon) = c$. These are scalars (real numbers and random variables), *not matrices.*

Calculate $Var(Y)$. Show your work and simplify. You are strongly encouraged to use the Centering Rule, but you can use the definition of a variance on the formula sheet if you really insist. Circle your final answer.

$$Var(Y) = Var(\overset{c}{Y}) = E\{(\beta_1 \overset{c}{X} + \varepsilon)^2\}$$

$$= E\{\beta_1^2 \overset{c}{X}{}^2 + 2\beta_1 \overset{c}{X}\varepsilon + \varepsilon^2\}$$

$$= \beta_1^2 E\{\overset{c}{X}{}^2\} + 2\beta_1 E\{\overset{c}{X}\varepsilon\} + E\{\varepsilon^2\}$$

$$= \beta_1^2 \sigma_x^2 + 2\beta_1 c + \sigma_\varepsilon^2$$