STA 431s17 Formulas[1]

$Var(X) = E\left\{(X-\mu_x)^2\right\} = E(X^2)-\mu_x^2$

$Cov(X,Y) = E\left\{(X-\mu_x)(Y-\mu_y)\right\} = E(XY)-\mu_x\mu_y$

$Corr(X,Y) = \rho_{xy} = \frac{Cov(X,Y)}{\sigma_x\sigma_y}$

$r_{xy} = \frac{\sum_{i=1}^n (x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum_{i=1}^n (x_i-\overline{x})^2}\sqrt{\sum_{i=1}^n (y_i-\overline{y})^2}}$

$cov(\mathbf{X}) = E\left\{(\mathbf{X}-\boldsymbol{\mu}_x)(\mathbf{X}-\boldsymbol{\mu}_x)^\top\right\}$

$cov(\mathbf{X},\mathbf{Y}) = E\left\{(\mathbf{X}-\boldsymbol{\mu}_x)(\mathbf{Y}-\boldsymbol{\mu}_y)^\top\right\}$

$cov(\mathbf{AX}) = \mathbf{A}\boldsymbol{\Sigma}_x\mathbf{A}^\top$

$cov(\mathbf{AX},\mathbf{BX}) = \mathbf{A}\boldsymbol{\Sigma}_x\mathbf{B}^\top$

$\mathbf{L} = \mathbf{A}_1\mathbf{X}_1 + \cdots + \mathbf{A}_m\mathbf{X}_m + \mathbf{b}$

$\stackrel{c}{\mathbf{L}} = \mathbf{A}_1 \stackrel{c}{\mathbf{X}}_1 + \cdots + \mathbf{A}_m \stackrel{c}{\mathbf{X}}_m$

$cov(\mathbf{L}) = E(\stackrel{c}{\mathbf{L}}\stackrel{c}{\mathbf{L}}^\top)$

$cov(\mathbf{L}_1,\mathbf{L}_2) = E(\stackrel{c}{\mathbf{L}}_1 \stackrel{c}{\mathbf{L}}_2^\top)$

$f(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$

$f(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{1}{|\boldsymbol{\Sigma}|^{\frac{1}{2}}(2\pi)^{\frac{p}{2}}}\exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$

If $\mathbf{X}\sim N(\boldsymbol{\mu},\boldsymbol{\Sigma})$, then $\mathbf{AX}+\mathbf{b}\sim N_p(\mathbf{A}\boldsymbol{\mu}+\mathbf{b},\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$.

$L(\boldsymbol{\mu},\boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-np/2}\exp-\frac{n}{2}\left\{tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{x}}-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\overline{\mathbf{x}}-\boldsymbol{\mu})\right\}$

$\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^n(\mathbf{x}_i-\overline{\mathbf{x}})(\mathbf{x}_i-\overline{\mathbf{x}})^\top$

$G^2 = -2\ln\left(\frac{\max_{\theta\in\Theta_0}L(\theta)}{\max_{\theta\in\Theta}L(\theta)}\right) = -2\ln\left(\frac{L(\widehat{\theta}_0)}{L(\widehat{\theta})}\right)$

If $W = X + e$,

Reliability is $Corr(W,X)^2 = \frac{\sigma_x^2}{\sigma_x^2+\sigma_e^2}$

The Double Measurement Model in centered form:

$\mathbf{Y}_i = \boldsymbol{\beta}\mathbf{X}_i + \boldsymbol{\epsilon}_i$

$cov(\mathbf{X}_i) = \boldsymbol{\Phi}_x$, $cov(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$

$\mathbf{F}_i = \left(\begin{array}{c}\mathbf{X}_i \\ \mathbf{Y}_i\end{array}\right)$

$\mathbf{X}_i$ is $p\times 1$, $\mathbf{Y}_i$ is $q\times 1$, $\mathbf{F}_i$ is $(p+q)\times 1$

$cov(\mathbf{F}_i) = \boldsymbol{\Phi}$

$\mathbf{D}_{i,1} = \mathbf{F}_i + \mathbf{e}_{i,1}$

$cov(\mathbf{e}_{i,1}) = \boldsymbol{\Omega}_1$, $cov(\mathbf{e}_{i,2}) = \boldsymbol{\Omega}_2$

$\mathbf{D}_{i,2} = \mathbf{F}_i + \mathbf{e}_{i,2}$

$\mathbf{X}_i$, $\boldsymbol{\epsilon}_i$, $\mathbf{e}_{i,1}$ and $\mathbf{e}_{i,2}$ are independent.

The General Structural Equation Model in centered form:

$\mathbf{Y}_i = \boldsymbol{\beta}\mathbf{Y}_i + \boldsymbol{\Gamma}\mathbf{X}_i + \boldsymbol{\epsilon}_i$

$cov(\mathbf{X}_i) = \boldsymbol{\Phi}_x$ and $cov(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$

$\mathbf{F}_i = \left(\begin{array}{c}\mathbf{X}_i \\ \mathbf{Y}_i\end{array}\right)$

$cov(\mathbf{F}_i) = \boldsymbol{\Phi} = \left(\begin{array}{cc}\boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \boldsymbol{\Phi}_{12}^\top & \boldsymbol{\Phi}_{22}\end{array}\right)$

$\mathbf{D}_i = \boldsymbol{\Lambda}\mathbf{F}_i + \mathbf{e}_i$

$cov(\mathbf{e}_i) = \boldsymbol{\Omega}$

$\mathbf{X}_i$, $\boldsymbol{\epsilon}_i$ and $\mathbf{e}_i$ are independent.

$\mathbf{X}_i$ is $p\times 1$, $\mathbf{Y}_i$ is $q\times 1$, $\mathbf{D}_i$ is $k\times 1$.

$\boldsymbol{\Phi}_x$ and $\boldsymbol{\Psi}$ are positive definite.
