

STA 431s17 Formulas¹

$$\text{Var}(X) = E\{(X - \mu_x)^2\} = E(X^2) - \mu_x^2 \quad \text{Cov}(X, Y) = E\{(X - \mu_x)(Y - \mu_y)\} = E(XY) - \mu_x\mu_y$$

$$\text{Corr}(X, Y) = \rho_{xy} = \frac{\text{Cov}(X, Y)}{\sigma_x\sigma_y} \quad r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

$$\text{cov}(\mathbf{X}) = E\{(\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{X} - \boldsymbol{\mu}_x)^\top\} \quad \text{cov}(\mathbf{X}, \mathbf{Y}) = E\{(\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{Y} - \boldsymbol{\mu}_y)^\top\}$$

$$\text{cov}(\mathbf{AX}) = \mathbf{A}\boldsymbol{\Sigma}_x\mathbf{A}^\top \quad \text{cov}(\mathbf{AX}, \mathbf{BX}) = \mathbf{A}\boldsymbol{\Sigma}_x\mathbf{B}^\top$$

$$\mathbf{L} = \mathbf{A}_1\mathbf{X}_1 + \cdots + \mathbf{A}_m\mathbf{X}_m + \mathbf{b} \quad \overset{c}{\mathbf{L}} = \mathbf{A}_1 \overset{c}{\mathbf{X}}_1 + \cdots + \mathbf{A}_m \overset{c}{\mathbf{X}}_m$$

$$\text{cov}(\mathbf{L}) = E(\overset{c}{\mathbf{L}}\overset{c}{\mathbf{L}}^\top) \quad \text{cov}(\mathbf{L}_1, \mathbf{L}_2) = E(\overset{c}{\mathbf{L}}_1 \overset{c}{\mathbf{L}}_2^\top)$$

$$f(x|\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\} \quad f(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{|\boldsymbol{\Sigma}|^{\frac{1}{2}}(2\pi)^{\frac{p}{2}}} \exp\left\{-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu})\right\}$$

If $\mathbf{X} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\mathbf{AX} + \mathbf{b} \sim N_p(\mathbf{A}\boldsymbol{\mu} + \mathbf{b}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$.

$$L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2} (2\pi)^{-np/2} \exp\left\{-\frac{n}{2}\left\{\text{tr}(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\bar{\mathbf{x}} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\bar{\mathbf{x}} - \boldsymbol{\mu})\right\}\right\}$$

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{n} \sum_{i=1}^n (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^\top \quad G^2 = -2 \ln \left(\frac{\max_{\theta \in \Theta_0} L(\theta)}{\max_{\theta \in \Theta} L(\theta)} \right) = -2 \ln \left(\frac{L(\hat{\theta}_0)}{L(\hat{\theta})} \right)$$

If $W = X + e$,

$$\text{Reliability is } \text{Corr}(W, X)^2 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_e^2}$$

The Double Measurement Model in centered form:

$$\mathbf{Y}_i = \boldsymbol{\beta}\mathbf{X}_i + \boldsymbol{\epsilon}_i$$

$$\text{cov}(\mathbf{X}_i) = \boldsymbol{\Phi}_x, \text{cov}(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$$

$$\mathbf{F}_i = \begin{pmatrix} \mathbf{X}_i \\ \mathbf{Y}_i \end{pmatrix}$$

\mathbf{X}_i is $p \times 1$, \mathbf{Y}_i is $q \times 1$, \mathbf{F}_i is $(p+q) \times 1$

$$\text{cov}(\mathbf{F}_i) = \boldsymbol{\Phi}$$

$$\mathbf{D}_{i,1} = \mathbf{F}_i + \mathbf{e}_{i,1}$$

$$\text{cov}(\mathbf{e}_{i,1}) = \boldsymbol{\Omega}_1, \text{cov}(\mathbf{e}_{i,2}) = \boldsymbol{\Omega}_2$$

$$\mathbf{D}_{i,2} = \mathbf{F}_i + \mathbf{e}_{i,2}$$

\mathbf{X}_i , $\boldsymbol{\epsilon}_i$, $\mathbf{e}_{i,1}$ and $\mathbf{e}_{i,2}$ are independent.

The General Structural Equation Model in centered form:

$$\mathbf{Y}_i = \boldsymbol{\beta}\mathbf{Y}_i + \boldsymbol{\Gamma}\mathbf{X}_i + \boldsymbol{\epsilon}_i$$

$$\text{cov}(\mathbf{X}_i) = \boldsymbol{\Phi}_x \text{ and } \text{cov}(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$$

$$\mathbf{F}_i = \begin{pmatrix} \mathbf{X}_i \\ \mathbf{Y}_i \end{pmatrix}$$

$$\text{cov}(\mathbf{F}_i) = \boldsymbol{\Phi} = \begin{pmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \boldsymbol{\Phi}_{12}^\top & \boldsymbol{\Phi}_{22} \end{pmatrix}$$

$$\mathbf{D}_i = \boldsymbol{\Lambda}\mathbf{F}_i + \mathbf{e}_i$$

$$\text{cov}(\mathbf{e}_i) = \boldsymbol{\Omega}$$

\mathbf{X}_i , $\boldsymbol{\epsilon}_i$ and \mathbf{e}_i are independent.

\mathbf{X}_i is $p \times 1$, \mathbf{Y}_i is $q \times 1$, \mathbf{D}_i is $k \times 1$.

$\boldsymbol{\Phi}_x$ and $\boldsymbol{\Psi}$ are positive definite.

¹This formula sheet was prepared by Jerry Brunner, Department of Statistics, University of Toronto. It is licensed under a [Creative Commons Attribution - ShareAlike 3.0 Unported License](https://creativecommons.org/licenses/by-sa/3.0/). Use any part of it as you like and share the result freely. The L^AT_EX source code is available from the course website: <http://www.utstat.toronto.edu/~brunner/oldclass/431s17>