NAME (PRINT): ____
Last/Surname First /Given Name

STUDENT #: ______________________ **SIGNATURE:** ______________________

UNIVERSITY OF TORONTO MISSISSAUGA
APRIL 2013 FINAL EXAMINATION
STA431H5S
Structural Equation Models
Jerry Brunner
Duration - 3 hours
Aids: Calculator Model(s): Any calculator is okay. Formula sheet supplied

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Please note, you ***CANNOT*** *petition to* ***re-write*** *an examination once the exam has begun.*

Qn. #	Value	Score
1	5	
2	5	
3	12	
4	12	
5	18	
6	12	
7	10	
8	6	
9	6	
10	14	
Total = 100 Points		

5 points

1. Let Let $\mathbf{X}$ be a $p \times 1$ random vector with mean $\boldsymbol{\mu}_x$ and variance-covariance matrix $\boldsymbol{\Sigma}_x$, and let $\mathbf{Y}$ be a $q \times 1$ random vector with mean $\boldsymbol{\mu}_y$ and variance-covariance matrix $\boldsymbol{\Sigma}_y$. Find $C(\mathbf{X}+\mathbf{c},\mathbf{Y}+\mathbf{d})$, where $\mathbf{c}$ and $\mathbf{d}$ are vectors of constants. Use the definition from the formula sheet. Do *not* use the centering rule directly.

   You have a lot more room than you need.

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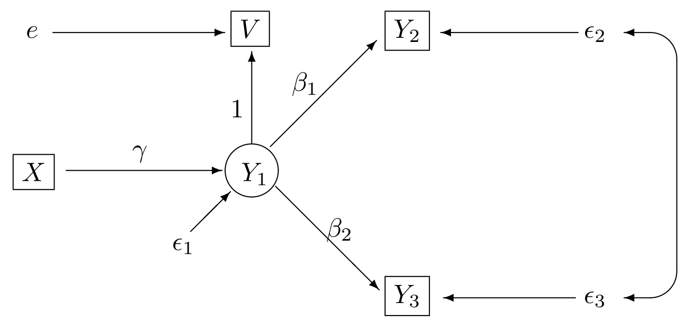
5 points

2. Let X_1 and X_2 be independent normal random variables, both having mean zero and variance one. Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_2$. Find the joint distribution of Y_1 and Y_2. Show your work. *Finish your answer with "So the joint distribution of Y_1 and Y_2 is ...*

continued on page 4

12 points

3. In the following model, all random variables are normally distributed with expected value zero, and there are no intercepts.



(a) Write the model equations in scalar form.

(b) What is the parameter vector $\boldsymbol{\theta}$ for this model? Use standard notation. Include unknown parameters only.

(c) Does this model pass the test of the parameter count rule? Answer Yes or No and give both numbers.

continued on page 5

12 points

4. Patients with high blood pressure are randomly assigned to different dosages of a blood pressure medication. There are lots of different dosages, so dosage may be treated as a continuous variable. Because the exact dosage is known, this variable is observed without error. After one month of taking the medication, the level of the drug in the patient's bloodstream is measured once (with error, of course), by an independent lab. Then, three independent measurements of the patient's blood pressure are taken. One is done by the lab that did the blood test, one is the average of 7 daily measurements taken at home by the patient, and one is done in the doctor's office. Notice that the same lab measures the blood level of the drug, and also does one of the blood pressure measurements. Do *not* assume that errors in the two measurements carried out by the lab are independent. **Make a path diagram. Do not bother to write coefficients on the arrows this time**, but write brief labels ("Dose" etc.) in the boxes and ovals.

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18 points

5. Consider the simple confirmatory factor analysis model

$$\begin{aligned} D_1 &= F_1 + e_1 \\ D_2 &= \lambda_2 F_1 + e_2 \\ D_3 &= F_2 + e_3 \\ D_4 &= \lambda_4 F_2 + e_4 \end{aligned}$$

where all expected values are zero, $Var(e_i) = \omega_i$ for $i = 1, \ldots, 4$, the error terms are independent of F_1 and F_2 and of each other,

$$V \left[\begin{array}{c} F_1 \\ F_2 \end{array} \right] = \left[\begin{array}{cc} \phi_{11} & \phi_{12} \\ \phi_{12} & \phi_{22} \end{array} \right],$$

with $\phi_{12} \neq 0$ (important), and λ_2 and λ_4 are nonzero constants.

(a) Give the covariance matrix of the observable variables. You do not need to show your work.

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(b) Are all the model parameters identifiable in Question 5? Answer Yes or No and prove your answer.

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12 points

6. Starting with the general two-stage model on the formula sheet, write the matrix $\mathbf{\Phi}$ in terms of the parameter matrices $\boldsymbol{\beta}$, $\mathbf{\Gamma}$, $\mathbf{\Phi}_{11}$ and $\mathbf{\Psi}$. Show your work. The final answer is a partitioned matrix. **Circle your final answer.**

continued on page 9

10 points

7. The following model has all expected values zero. The covariance between X_1 and X_2 is ϕ_{12}, while the error terms have zero covariance with one another and with X_1 and X_2. Only the variables W_1, W_2, V_1 and V_2 are observable.

$$
\begin{aligned}
Y_1 &= \gamma_1 X_1 + \gamma_2 X_2 + \epsilon_1 \\
Y_2 &= \beta Y_1 + \gamma_3 X_1 + \epsilon_2 \\
W_1 &= \lambda_1 X_1 + e_1 \\
W_2 &= \lambda_2 X_2 + e_2 \\
V_1 &= \lambda_3 Y_1 + e_3 \\
V_2 &= \lambda_4 Y_2 + e_4
\end{aligned}
$$

Make a path diagram. Where you do *not* write a coefficient on an arrow (or double-headed arrow), it means the coefficient equals one (this time).

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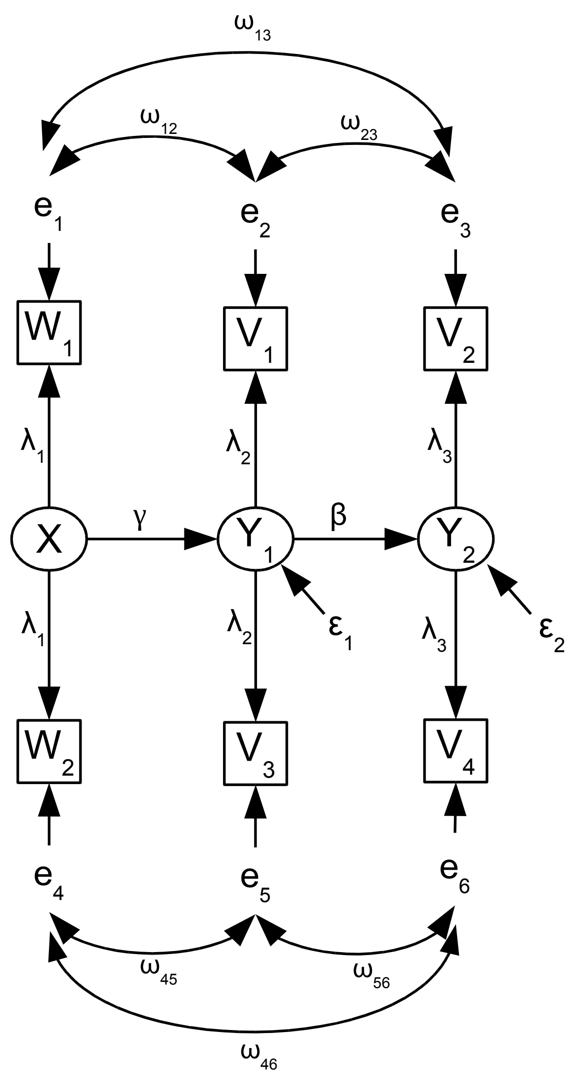
6 points

8. The following path diagram represents an *original* model; its parameters are not identifiable (you don't have to prove this). Notice that some of the factor loadings are the same. This is a modeling restriction, and reflects the fact that some measurement procedures were very similar.



Give a set of restrictions on the parameters that make the parameters of the *re-parameterized* model identifiable. The fewer restrictions you specify, the better. You don't have to justify your answer.

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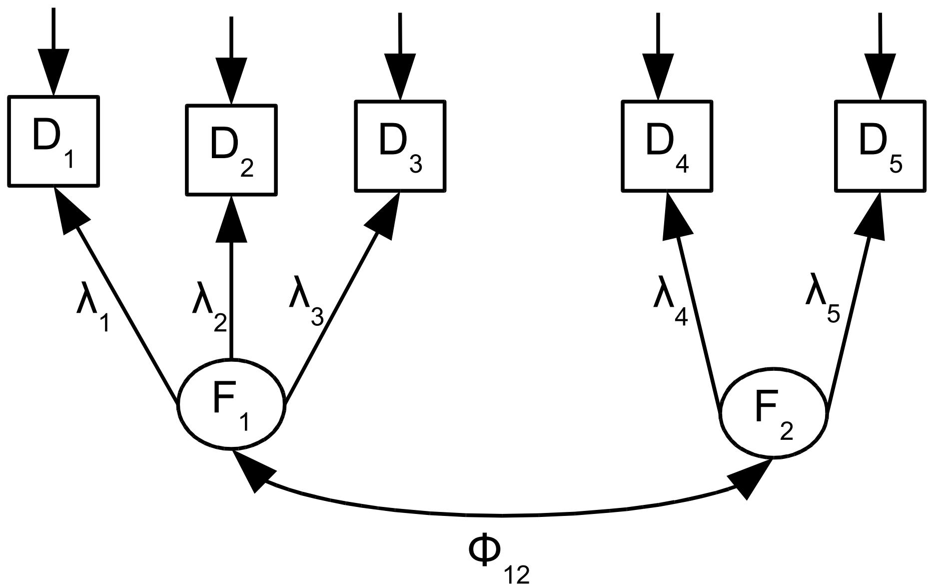
6 points

9. The factor analysis model pictured below has been re-parameterized by setting the variances of both factors to one.



What three additional conditions are needed for all the parameters of this re-parameterized model to be identifiable? You don't have to justify your answer. Just give three conditions that make the parameters identifiable.

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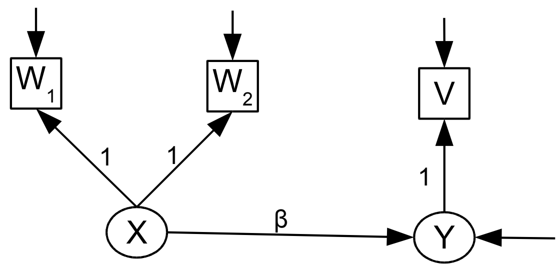
14 points 10. For the following model,



the sample variance-covariance matrix $\widehat{\Sigma}$ is

```
      W1    W2     V
W1 38.53 21.39 19.85
W2 21.39 35.50 19.00
V  19.85 19.00 28.81
```

Give a reasonable estimate of β. There is more than one right answer. Show your work. There are some calculations involving symbols, but the answer is a number. **Circle your answer.**