STA 431s15 Formulas[1]

Columns of $\mathbf{A}$ *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{A}\mathbf{v} = \mathbf{0}$.

$\mathbf{A}$ *positive definite* means $\mathbf{v}^\top \mathbf{A}\mathbf{v} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$E(g(X)) = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx,$ or $E(g(X)) = \sum_x g(x)\, p_X(x)$

$Var(X) = E[(X - \mu_X)^2]$

$Cov(X,Y) = E[(X - \mu_X)(Y - \mu_Y)]$

$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$

$r = \frac{\sum_{i=1}^n (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^n (X_i - \overline{X})^2 \sum_{i=1}^n (Y_i - \overline{Y})^2}}$

$V(\mathbf{X}) = E\left\{(\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{X} - \boldsymbol{\mu}_x)^\top\right\}$

$C(\mathbf{X},\mathbf{Y}) = E\left\{(\mathbf{X} - \boldsymbol{\mu}_x)(\mathbf{Y} - \boldsymbol{\mu}_y)^\top\right\}$

$\mathbf{L} = \mathbf{A}_1\mathbf{X}_1 + \cdots + \mathbf{A}_m\mathbf{X}_m + \mathbf{b}$

$\overset{c}{\mathbf{L}} = \mathbf{A}_1 \overset{c}{\mathbf{X}}_1 + \cdots + \mathbf{A}_m \overset{c}{\mathbf{X}}_m$

$V(\mathbf{L}) = E(\overset{c}{\mathbf{L}}\overset{c}{\mathbf{L}}^\top)$

$C(\mathbf{L}_1, \mathbf{L}_2) = E(\overset{c}{\mathbf{L}}_1 \overset{c}{\mathbf{L}}_2^\top)$

$f(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$

$f(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{1}{|\boldsymbol{\Sigma}|^{\frac{1}{2}}(2\pi)^{\frac{p}{2}}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$

If $\mathbf{X} \sim N(\boldsymbol{\mu},\boldsymbol{\Sigma})$, then $\mathbf{A}\mathbf{X} + \mathbf{b} \sim N_p(\mathbf{A}\boldsymbol{\mu} + \mathbf{b}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$.

$L(\boldsymbol{\mu},\boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-np/2} \exp -\frac{n}{2}\left\{tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{x}}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\overline{\mathbf{x}}-\boldsymbol{\mu})\right\}$

$\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^n (\mathbf{x}_i - \overline{\mathbf{x}})(\mathbf{x}_i - \overline{\mathbf{x}})^\top$

$G^2 = -2\ln\left(\frac{\max_{\theta\in\Theta_0} L(\theta)}{\max_{\theta\in\Theta} L(\theta)}\right) = -2\ln\left(\frac{L(\widehat{\theta}_0)}{L(\widehat{\theta})}\right)$

If $W = X + e$, Reliability is $Corr(W,X)^2 = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_e^2}$

The Double Measurement Model in centered form:

$\mathbf{Y}_i = \boldsymbol{\beta}\mathbf{X}_i + \boldsymbol{\epsilon}_i$

$V(\mathbf{X}_i) = \boldsymbol{\Phi}_x$, $V(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$

$\mathbf{F}_i = \begin{pmatrix} \mathbf{X}_i \\ \mathbf{Y}_i \end{pmatrix}$

$\mathbf{X}_i$ is $p \times 1$, $\mathbf{Y}_i$ is $q \times 1$, $\mathbf{F}_i$ is $(p+q) \times 1$

$V(\mathbf{F}_i) = \boldsymbol{\Phi}$

$\mathbf{D}_{i,1} = \mathbf{F}_i + \mathbf{e}_{i,1}$

$V(\mathbf{e}_{i,1}) = \boldsymbol{\Omega}_1$, $V(\mathbf{e}_{i,2}) = \boldsymbol{\Omega}_2$

$\mathbf{D}_{i,2} = \mathbf{F}_i + \mathbf{e}_{i,2}$

$\mathbf{X}_i$, $\boldsymbol{\epsilon}_i$, $\mathbf{e}_{i,1}$ and $\mathbf{e}_{i,2}$ are independent.

The General Structural Equation Model in centered form:

$\mathbf{Y}_i = \boldsymbol{\beta}\mathbf{Y}_i + \boldsymbol{\Gamma}\mathbf{X}_i + \boldsymbol{\epsilon}_i$

$V(\mathbf{X}_i) = \boldsymbol{\Phi}_x$ and $V(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$

$\mathbf{F}_i = \begin{pmatrix} \mathbf{X}_i \\ \mathbf{Y}_i \end{pmatrix}$

$V(\mathbf{F}_i) = \boldsymbol{\Phi} = \begin{pmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \boldsymbol{\Phi}_{12}^\top & \boldsymbol{\Phi}_{22} \end{pmatrix}$

$\mathbf{D}_i = \boldsymbol{\Lambda}\mathbf{F}_i + \mathbf{e}_i$

$V(\mathbf{e}_i) = \boldsymbol{\Omega}$

$\mathbf{X}_i$, $\boldsymbol{\epsilon}_i$ and $\mathbf{e}_i$ are independent.

$\mathbf{X}_i$ is $p \times 1$, $\mathbf{Y}_i$ is $q \times 1$, $\mathbf{D}_i$ is $k \times 1$.

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[1] This formula sheet was prepared by Jerry Brunner, Department of Statistics, University of Toronto. The LaTeX source code is available from the course website: http://www.utstat.toronto.edu/~brunner/oldclass/431s15