Structural Equation Models

An extension of multiple regression.
Can incorporate measurement error,
and more

Full generality later

First, introduction to multiple regression with measurement error

Linear Regression

$$Y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \cdots + \beta_{p-1} x_{i,p-1} + \epsilon_i$$

where $\epsilon_1, \ldots \epsilon_n$ are independent random variables with expected value zero and common variance σ^2, and $x_{i,1}, \ldots x_{i,p-1}$ are fixed constants.

Matrix Form

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

where $\mathbf{X}$ is an $n \times p$ matrix of known constants, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown constants, and $\boldsymbol{\epsilon}$ is multivariate normal with mean zero and covariance matrix $\sigma^2 \mathbf{I}_n$

Are *X* values really constants?

Double Expectation

$$E\{Y\} = E\{E\{Y|X\}\}$$

E{Y} is a constant. E{Y|X} is a random variable, a function of X.

$$E\{E\{Y|X\}\} = \int E\{Y|X = x\}\, f(x)\, dx$$

Beta-hat is (conditionally) unbiased

$$E\{\widehat{\boldsymbol{\beta}}|\mathbf{X} = \mathbf{x}\} = \boldsymbol{\beta}$$

Unbiased unconditionally, too

$$E\{\widehat{\boldsymbol{\beta}}\} = E\{E\{\widehat{\boldsymbol{\beta}}|\mathbf{X} = \mathbf{x}\}\} = E\{\boldsymbol{\beta}] = \boldsymbol{\beta}$$

Perhaps Clearer

$$\begin{aligned}
E\{\widehat{\boldsymbol{\beta}}\} &= E\{E\{\widehat{\boldsymbol{\beta}}|\mathbf{X}=\mathbf{x}\}\} \\
&= \int \cdots \int E\{\widehat{\boldsymbol{\beta}}|\mathbf{X}=\mathbf{x}\}\, f(\mathbf{x})\, d\mathbf{x} \\
&= \int \cdots \int \boldsymbol{\beta}\, f(\mathbf{x})\, d\mathbf{x} \\
&= \boldsymbol{\beta} \int \cdots \int f(\mathbf{x})\, d\mathbf{x} \\
&= \boldsymbol{\beta} \cdot 1 = \boldsymbol{\beta}.
\end{aligned}$$

Conditional size alpha test, Critical region *A*

$$Pr\{F \in A | \mathbf{X} = \mathbf{x}\} = \alpha$$

$$\begin{aligned} Pr\{F \in A\} &= \int \cdots \int \alpha f(\mathbf{x})\, d\mathbf{x} \\ &= \alpha \int \cdots \int f(\mathbf{x})\, d\mathbf{x} \\ &= \alpha. \end{aligned}$$