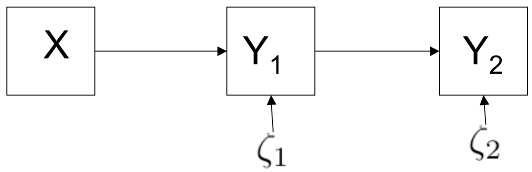
Estimation and Testing



$$
\begin{aligned}
Y_1 &= \gamma X + \zeta_1 \\
Y_2 &= \beta Y_1 + \zeta_2
\end{aligned}
$$

$E(X) = E(\zeta_1) = E(\zeta_2) = 0$

$V(X) = \phi, V(\zeta_1) = \psi_1, V(\zeta_2) = \psi_2$

X, ζ_1, ζ_2 are independent

Everything is normal

Distribution of the data

$\left[\begin{array}{c} X_1 \\ Y_{1,1} \\ Y_{1,2} \end{array}\right] \ldots \left[\begin{array}{c} X_n \\ Y_{n,1} \\ Y_{n,2} \end{array}\right]$ are independent normal with mean zero

and covariance matrix

$$
\boldsymbol{\Sigma} = \left[\begin{array}{ccc}
\phi & \gamma\phi & \beta\gamma\phi \\
\gamma\phi & \gamma^2\phi + \psi_1 & \beta(\gamma^2\phi + \psi_1) \\
\beta\gamma\phi & \beta(\gamma^2\phi + \psi_1) & \beta^2(\gamma^2\phi + \psi_1) + \psi_2
\end{array}\right]
$$

$$
\boldsymbol{\theta} = (\gamma, \beta, \phi, \psi_1, \psi_2)
$$

Maximum Likelihood

$$
L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-nk/2} \exp -\frac{n}{2}\left[tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{x}} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\overline{\mathbf{x}} - \boldsymbol{\mu})\right]
$$

Minimize $-2\ell(\boldsymbol{\mu}(\boldsymbol{\theta}), \boldsymbol{\Sigma}(\boldsymbol{\theta}))$

$$
\begin{aligned}
= n\left[\log|\boldsymbol{\Sigma}(\boldsymbol{\theta})| + k\log(2\pi) + tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1})\right. \\
\left. + (\overline{\mathbf{x}} - \boldsymbol{\mu}(\boldsymbol{\theta}))'\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1}(\overline{\mathbf{x}} - \boldsymbol{\mu}(\boldsymbol{\theta}))\right]
\end{aligned}
$$

Likelihood Ratio Test for Goodness of Fit

$$
\begin{aligned}
G &= -2\log\frac{L(\overline{\mathbf{X}}, \boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}}))}{L(\overline{\mathbf{X}}, \widehat{\boldsymbol{\Sigma}})} \\
&= n[\log|\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}})| + k\log(2\pi) + tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}})^{-1}] \\
&\quad -n[\log|\widehat{\boldsymbol{\Sigma}}| + k\log(2\pi) + k] \\
&= n[\log|\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}})| - \log|\widehat{\boldsymbol{\Sigma}}| + tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}(\widehat{\boldsymbol{\theta}})^{-1} - k]
\end{aligned}
$$

Do it all at once: Minimize

$$G(\boldsymbol{\theta}) = n[\log|\boldsymbol{\Sigma}(\boldsymbol{\theta})| - \log|\widehat{\boldsymbol{\Sigma}}| + tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1}) - k]$$

Actually, SAS minimizes the "Objective Function"

$$\log|\boldsymbol{\Sigma}(\boldsymbol{\theta})| - \log|\widehat{\boldsymbol{\Sigma}}| + tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}(\boldsymbol{\theta})^{-1}) - k$$

Chi-square and Z Tests

- "Chisquare" is (n-1) times minimum objective function.
- Test nested models by difference between chi-square values
- Z tests are produced by default; Asymptotic Covariance matrix is available

- Likelihood ratio tests perform better