More Confirmatory Factor Analyses of the Twin Data

Recall from Handout 6: A two-factor model did not fit.

```
Chi-Square                                          143.0653
Chi-Square DF                                             26
Pr > Chi-Square                                       <.0001
```

But we determined that a model with a third factor (measured by height and weight) distinct from the head size variables, would be identified if the covariance between the body size and head size factors were non-zero. First, I dropped Bizygomatic Breadth and tried to fit a two-factor model, but got *a negative variance estimate* – more on this later. Now try three factors.

```
%include 'twinread.sas';
TITLE2 'Confirmatory Factor Analysis: Try 2';

proc calis corr residual; /* Residuals are corr - reproduced */
     title3 'Try 3 factors';
     var progmat reason verbal     /* Name the observed vars */
         headlng headbrd headcir bizyg height weight;
     lineqs
         progmat = lambda1 F1 + e1,
         reason  = lambda2 F1 + e2,
         verbal  = lambda3 F1 + e3,
         headlng = lambda4 F2 + e4,
         headbrd = lambda5 F2 + e5,
         headcir = lambda6 F2 + e6,
         bizyg   = lambda7 F2 + e7,
         height  = lambda8 F3 + e8,
         weight  = lambda9 F3 + e9;
     std     /* Variances (not standard deviations) of exogenous vars */
         F1 F2 F3 = 3 * 1, e1-e9 = 9 * psi:;
         /* Don't forget the colon :  */
     cov F1 F2 = phi12, F1 F3 = phi13, F2 F3 = phi23;
     bounds 0 < psi1-psi9;
```

Skipping some familiar parts of the output ...

```
                        The 9 Endogenous Variables

    Manifest        progmat  reason   verbal   headlng  headbrd  headcir
                    bizyg    height   weight
    Latent


                        The 12 Exogenous Variables

    Manifest
    Latent          F1       F2       F3
    Error           e1       e2       e3       e4       e5       e6
                    e7       e8       e9
```

Correlations

	progmat	reason	verbal	headlng	headbrd
progmat	1.0000	0.5503	0.6136	0.3138	0.1609
reason	0.5503	1.0000	0.7537	0.1582	0.0728
verbal	0.6136	0.7537	1.0000	0.2777	0.1467
headlng	0.3138	0.1582	0.2777	1.0000	0.3002
headbrd	0.1609	0.0728	0.1467	0.3002	1.0000
headcir	0.3314	0.2589	0.3149	0.8336	0.6775
bizyg	0.1843	0.2173	0.2473	0.4541	0.8046
height	0.2840	0.1403	0.2241	0.5915	0.4609
weight	0.2290	0.1452	0.1557	0.5695	0.4942

Correlations

	headcir	bizyg	height	weight
progmat	0.3314	0.1843	0.2840	0.2290
reason	0.2589	0.2173	0.1403	0.1452
verbal	0.3149	0.2473	0.2241	0.1557
headlng	0.8336	0.4541	0.5915	0.5695
headbrd	0.6775	0.8046	0.4609	0.4942
headcir	1.0000	0.7247	0.6115	0.6915
bizyg	0.7247	1.0000	0.6631	0.6570
height	0.6115	0.6631	1.0000	0.6677
weight	0.6915	0.6570	0.6677	1.0000

Determinant 0.000831 Ln -7.092601

Confirmatory Factor Analysis: Try 2
Try 3 factors

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Levenberg-Marquardt Optimization

Scaling Update of More (1978)

Parameter Estimates	21
Functions (Observations)	45
Lower Bounds	9
Upper Bounds	0

Optimization Start

Active Constraints	0	Objective Function	2.0237621863
Max Abs Gradient Element	1.3904154744	Radius	7.8123643598

Iter	Rest arts	Func Calls	Act Con	Objective Function	Obj Fun Change	Max Abs Gradient Element	Lambda	Actual Over Pred Change
1	0	2	1	1.78018	0.2436	0.3653	0	1.025
2	0	3	1	1.76567	0.0145	0.00859	0	1.068
3	0	4	1	1.76564	0.000038	0.00180	0	1.080
4	0	5	1	1.76564	6.665E-7	0.000162	0	1.107
5	0	6	1	1.76564	1.418E-8	0.000038	0	1.112
6	0	7	1	1.76564	3.04E-10	3.422E-6	0	1.115

Optimization Results

Iterations	6	Function Calls	8
Jacobian Calls	7	Active Constraints	1
Objective Function	1.7656359425	Max Abs Gradient Element	3.421549E-6
Lambda	0	Actual Over Pred Change	1.1150982864
Radius	0.0000642188		

GCONV convergence criterion satisfied.

WARNING: There are 1 active constraints at the solution. The standard errors and Chi-Square test statistic assume the solution is located in the interior of the parameter space and hence do not apply if it is likely that some different set of inequality constraints could be active.

NOTE: The degrees of freedom are increased by the number of active constraints (see Dijkstra, 1992). The number of parameters in calculating fit indices is decreased by the number of active constraints. To turn off the adjustment, use the NOADJDF option.

The "active constraints" warning means that at least one of the estimated variances ran up against the boundary set by the `bounds` statement. It's trying to be negative! This is unacceptable, because it's outside the parameter space. We suspect this is because of correlated errors. Let's finish looking at the output, and then we'll see why.

Predicted Model Matrix

	progmat	reason	verbal	headlng	headbrd
progmat	1.0000	0.5546	0.6178	0.1983	0.1612
reason	0.5546	1.0000	0.7507	0.2410	0.1959
verbal	0.6178	0.7507	1.0000	0.2685	0.2182
headlng	0.1983	0.2410	0.2685	1.0000	0.5648
headbrd	0.1612	0.1959	0.2182	0.5648	1.0000
headcir	0.2379	0.2891	0.3220	0.8336	0.6775
bizyg	0.1724	0.2095	0.2334	0.6041	0.4910
height	0.1291	0.1568	0.1747	0.5080	0.4129
weight	0.1467	0.1782	0.1985	0.5773	0.4692

Predicted Model Matrix

	headcir	bizyg	height	weight
progmat	0.2379	0.1724	0.1291	0.1467
reason	0.2891	0.2095	0.1568	0.1782
verbal	0.3220	0.2334	0.1747	0.1985
headlng	0.8336	0.6041	0.5080	0.5773
headbrd	0.6775	0.4910	0.4129	0.4692
headcir	1.0000	0.7247	0.6095	0.6925
bizyg	0.7247	1.0000	0.4417	0.5019
height	0.6095	0.4417	1.0000	0.6677
weight	0.6925	0.5019	0.6677	1.0000

Determinant 0.004859 Ln -5.326965

Twin Data 6
Confirmatory Factor Analysis: Try 2
Try 3 factors

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Fit Function	1.7656
Goodness of Fit Index (GFI)	0.7691
GFI Adjusted for Degrees of Freedom (AGFI)	0.5845
Root Mean Square Residual (RMR)	0.0896
Parsimonious GFI (Mulaik, 1989)	0.5341
Chi-Square	128.8914
Chi-Square DF	25
Pr > Chi-Square	<.0001
Independence Model Chi-Square	517.76
Independence Model Chi-Square DF	36
RMSEA Estimate	0.2386
RMSEA 90% Lower Confidence Limit	0.1988
RMSEA 90% Upper Confidence Limit	0.2801

etc.

WARNING: The central parameter matrix _PHI_ has probably 1 zero eigenvalue(s).

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Raw Residual Matrix

	progmat	reason	verbal	headlng	headbrd
progmat	0.00000	-.00430	-.00422	0.11547	-.00025
reason	-.00430	0.00000	0.00304	-.08282	-.12306
verbal	-.00422	0.00304	0.00000	0.00925	-.07147
headlng	0.11547	-.08282	0.00925	0.00000	-.26458
headbrd	-.00025	-.12306	-.07147	-.26458	0.00000
headcir	0.09347	-.03023	-.00718	0.00000	0.00000
bizyg	0.01190	0.00776	0.01394	-.15003	0.31355
height	0.15488	-.01652	0.04943	0.08349	0.04796
weight	0.08234	-.03300	-.04283	-.00778	0.02504

Raw Residual Matrix

	headcir	bizyg	height	weight
progmat	0.09347	0.01190	0.15488	0.08234
reason	-.03023	0.00776	-.01652	-.03300
verbal	-.00718	0.01394	0.04943	-.04283
headlng	0.00000	-.15003	0.08349	-.00778
headbrd	0.00000	0.31355	0.04796	0.02504
headcir	0.00000	0.00000	0.00203	-.00105
bizyg	0.00000	0.00000	0.22138	0.15509
height	0.00203	0.22138	0.00000	0.00000
weight	-.00105	0.15509	0.00000	0.00000

Average Absolute Residual	0.049541
Average Off-diagonal Absolute Residual	0.061927

Rank Order of the 10 Largest Raw Residuals

Row	Column	Residual
bizyg	headbrd	0.31355
headbrd	headlng	-0.26458
height	bizyg	0.22138
weight	bizyg	0.15509
height	progmat	0.15488
bizyg	headlng	-0.15003
headbrd	reason	-0.12306
headlng	progmat	0.11547
headcir	progmat	0.09347
height	headlng	0.08349

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Asymptotically Standardized Residual Matrix

	progmat	reason	verbal	headlng	headbrd
progmat	0.0000	-0.2429	-0.5701	1.3083	-0.0026
reason	-0.2429	0.0000	1.1354	-1.1138	-1.3834
verbal	-0.5701	1.1354	0.0000	0.1450	-0.8631
headlng	1.3083	-1.1138	0.1450	0.0000	-6.1526
headbrd	-0.0026	-1.3834	-0.8631	-6.1526	0.0000
headcir	1.2481	-0.5984	-0.2925	0.0000	0.0000
bizyg	0.1252	0.0912	0.1788	-3.8570	5.5369
height	1.7129	-0.2173	0.7731	1.7415	0.7196
weight	1.0024	-0.5397	-1.0646	-0.1861	0.4189

Asymptotically Standardized Residual Matrix

	headcir	bizyg	height	weight
progmat	1.2481	0.1252	1.7129	1.0024
reason	-0.5984	0.0912	-0.2173	-0.5397
verbal	-0.2925	0.1788	0.7731	-1.0646
headlng	0.0000	-3.8570	1.7415	-0.1861
headbrd	0.0000	5.5369	0.7196	0.4189
headcir	0.0000	0.0000	0.8432	-0.8432
bizyg	0.0000	0.0000	3.5730	2.8036
height	0.8432	3.5730	0.0000	0.0000
weight	-0.8432	2.8036	0.0000	0.0000

Average Standardized Residual	0.917430
Average Off-diagonal Standardized Residual	1.146788

Rank Order of the 10 Largest Asymptotically Standardized Residuals

Row	Column	Residual
headbrd	headlng	-6.15256
bizyg	headbrd	5.53693
bizyg	headlng	-3.85701
height	bizyg	3.57300
weight	bizyg	2.80357
height	headlng	1.74149
height	progmat	1.71288
headbrd	reason	-1.38338
headlng	progmat	1.30828
headcir	progmat	1.24807

Distribution of Asymptotically Standardized Residuals

Each * Represents 1 Residuals

Range		Freq	Percent	
-6.25000	-6.00000	1	2.22	*
-6.00000	-5.75000	0	0.00	
-5.75000	-5.50000	0	0.00	
-5.50000	-5.25000	0	0.00	
-5.25000	-5.00000	0	0.00	
-5.00000	-4.75000	0	0.00	
-4.75000	-4.50000	0	0.00	
-4.50000	-4.25000	0	0.00	
-4.25000	-4.00000	0	0.00	
-4.00000	-3.75000	1	2.22	*
-3.75000	-3.50000	0	0.00	
-3.50000	-3.25000	0	0.00	
-3.25000	-3.00000	0	0.00	
-3.00000	-2.75000	0	0.00	
-2.75000	-2.50000	0	0.00	
-2.50000	-2.25000	0	0.00	
-2.25000	-2.00000	0	0.00	
-2.00000	-1.75000	0	0.00	
-1.75000	-1.50000	0	0.00	
-1.50000	-1.25000	1	2.22	*
-1.25000	-1.00000	2	4.44	**
-1.00000	-0.75000	2	4.44	**
-0.75000	-0.50000	3	6.67	***
-0.50000	-0.25000	1	2.22	*
-0.25000	0	4	8.89	****
0	0.25000	17	37.78	*****************
0.25000	0.50000	1	2.22	*
0.50000	0.75000	1	2.22	*
0.75000	1.00000	2	4.44	**
1.00000	1.25000	3	6.67	***
1.25000	1.50000	1	2.22	*
1.50000	1.75000	2	4.44	**
1.75000	2.00000	0	0.00	
2.00000	2.25000	0	0.00	
2.25000	2.50000	0	0.00	
2.50000	2.75000	0	0.00	
2.75000	3.00000	1	2.22	*
3.00000	3.25000	0	0.00	
3.25000	3.50000	0	0.00	
3.50000	3.75000	1	2.22	*
3.75000	4.00000	0	0.00	
4.00000	4.25000	0	0.00	
4.25000	4.50000	0	0.00	
4.50000	4.75000	0	0.00	
4.75000	5.00000	0	0.00	
5.00000	5.25000	0	0.00	
5.25000	5.50000	0	0.00	
5.50000	5.75000	1	2.22	*

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Manifest Variable Equations with Estimates

```
progmat =   0.6756*F1       +  1.0000 e1
Std Err     0.1091 lambda1
t Value     6.1925
reason  =   0.8209*F1       +  1.0000 e2
Std Err     0.1043 lambda2
t Value     7.8675
verbal  =   0.9145*F1       +  1.0000 e3
Std Err     0.1009 lambda3
t Value     9.0649
headlng =   0.8336*F2       +  1.0000 e4
Std Err     0.0945 lambda4
t Value     8.8164
headbrd =   0.6775*F2       +  1.0000 e5
Std Err     0.1027 lambda5
t Value     6.5948
headcir =   1.0000*F2       +  1.0000 e6
Std Err     0.0828 lambda6
t Value    12.0830
bizyg   =   0.7247*F2       +  1.0000 e7
Std Err     0.1005 lambda7
t Value     7.2110
height  =   0.7666*F3       +  1.0000 e8
Std Err     0.1064 lambda8
t Value     7.2072
weight  =   0.8710*F3       +  1.0000 e9
Std Err     0.1030 lambda9
t Value     8.4528
```

Variances of Exogenous Variables

Variable	Parameter	Estimate	Standard Error	t Value
F1		1.00000		
F2		1.00000		
F3		1.00000		
e1	psi1	0.54355	0.10259	5.30
e2	psi2	0.32610	0.08819	3.70
e3	psi3	0.16374	0.09008	1.82
e4	psi4	0.30515	0.05051	6.04
e5	psi5	0.54097	0.08954	6.04
e6	psi6	**0**	0	.
e7	psi7	0.47475	0.07858	6.04
e8	psi8	0.41238	0.09226	4.47
e9	psi9	0.24133	0.08954	2.70

Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate	Standard Error	t Value
F1	F2	phi12	0.35217	0.10986	3.21
F1	F3	phi13	0.24923	0.13013	1.92
F2	F3	phi23	0.79505	0.06018	13.21

I tried dropping head size variables one at a time, but always got one negative variance estimate.

```
/************ twinfac4.sas ********************/
%include 'twinread.sas';
TITLE2 'Confirmatory Factor Analysis with Double Measurement';

proc calis corr residual;
     title3 'Three Factors';
     var progmat1 reason1 verbal1     /* Name the observed vars */
         headlng1 headbrd1 headcir1 bizyg1 height1 weight1
         progmat2 reason2 verbal2
         headlng2 headbrd2 headcir2 bizyg2 height2 weight2;
     lineqs
         progmat1 = lambda1 F1 + e1,
         reason1  = lambda2 F1 + e2,
         verbal1  = lambda3 F1 + e3,
         headlng1 = lambda4 F2 + e4,
         headbrd1 = lambda5 F2 + e5,
         headcir1 = lambda6 F2 + e6,
         bizyg1   = lambda7 F2 + e7,
         height1  = lambda8 F3 + e8,
         weight1  = lambda9 F3 + e9,

         /* Specify equal parameter values by giving the same name. */

         progmat2 = lambda1 F1 + e10,
         reason2  = lambda2 F1 + e11,
         verbal2  = lambda3 F1 + e12,
         headlng2 = lambda4 F2 + e13,
         headbrd2 = lambda5 F2 + e14,
         headcir2 = lambda6 F2 + e15,
         bizyg2   = lambda7 F2 + e16,
         height2  = lambda8 F3 + e17,
         weight2  = lambda9 F3 + e18;

     std     /* Variances (not standard deviations) of exogenous vars.
                Be careful! It's easy to forget the colon :  */
         F1 F2 F3 = 3 * 1, e1-e18 = 18 * psi:;
     cov F1 F2 = phi12, F1 F3 = phi13, F2 F3 = phi23,
         e1-e9 = 36 * cova:, e10-e18 = 36 * covb:;
     bounds 0 < psi1-psi18, lambda1 > 0, lambda4 > 0, lambda8 > 0 ;
```

The good news is that all variance estimates are positive. There is a lot of output, partly because there are so many parameters. Being very selective ...

Predicted Model Matrix

		progmat1	reason1	verbal1
progmat1	Progressive matrices 1 (M)	1.0395	0.5672	0.5297
reason1	Reasoning Ability 1 (R)	0.5672	1.0429	0.7439
verbal1	Verbal Ability 1 (V)	0.5297	0.7439	1.0185
headlng1	Head Length 1 (L)	0.0952	0.0309	0.1075
headbrd1	Head breadth 1 (B)	0.0633	0.1230	0.1677
headcir1	Head Circumference 1 (C)	0.1389	0.1677	0.1959
bizyg1	Bizygomatic Breadth (Z): Dist betw eyes?	-0.0014	0.1531	0.1809

etc.

Average Absolute Residual	0.064079
Average Off-diagonal Absolute Residual	0.069296

WARNING: Fitting a correlation matrix should lead to insignificant diagonal residuals. The maximum value 0.0632339902 of the diagonal residuals may be too high for a valid Chi-Square test statistic and standard errors.

Reproduced diagonal elements of the correlation matrix are equal to one when everything is going well.

And the fit is awful.

Chi-Square	259.6440
Chi-Square DF	69
Pr > Chi-Square	<.0001

Rank Order of the 10 Largest Asymptotically Standardized Residuals

Row	Column	Residual
weight2	weight1	5.91855
headlng2	headlng1	5.41891
headbrd2	headbrd1	5.03520
headcir2	headcir1	4.18084
reason2	reason1	3.81390
headcir2	headlng1	3.48228
headlng2	headcir1	2.86252
height2	height1	2.56117
progmat2	progmat1	2.47306
headcir2	weight1	2.46035

Look at the top five! What do they have in common?

The CALIS Procedure
Covariance Structure Analysis: Maximum Likelihood Estimation

Distribution of Asymptotically Standardized Residuals

Each * Represents 1 Residuals

Range		Freq	Percent	
-2.00000	-1.75000	4	2.34	****
-1.75000	-1.50000	3	1.75	***
-1.50000	-1.25000	7	4.09	*******
-1.25000	-1.00000	7	4.09	*******
-1.00000	-0.75000	7	4.09	*******
-0.75000	-0.50000	15	8.77	***************
-0.50000	-0.25000	13	7.60	*************
-0.25000	0	21	12.28	*********************
0	0.25000	21	12.28	*********************
0.25000	0.50000	15	8.77	***************
0.50000	0.75000	13	7.60	*************
0.75000	1.00000	12	7.02	************
1.00000	1.25000	6	3.51	******
1.25000	1.50000	7	4.09	*******
1.50000	1.75000	4	2.34	****
1.75000	2.00000	3	1.75	***
2.00000	2.25000	3	1.75	***
2.25000	2.50000	2	1.17	**
2.50000	2.75000	1	0.58	*
2.75000	3.00000	1	0.58	*
3.00000	3.25000	0	0.00	
3.25000	3.50000	1	0.58	*
3.50000	3.75000	0	0.00	
3.75000	4.00000	1	0.58	*
4.00000	4.25000	1	0.58	*
4.25000	4.50000	0	0.00	
4.50000	4.75000	0	0.00	
4.75000	5.00000	0	0.00	
5.00000	5.25000	1	0.58	*
5.25000	5.50000	1	0.58	*
5.50000	5.75000	0	0.00	
5.75000	6.00000	1	0.58	*

Try a second-order factor analysis.

```
/************ twinfac5.sas ********************/
%include 'twinread.sas';
TITLE2 'Confirmatory Factor Analysis with Double Measurement';

proc calis corr residual;
     title3 'Second-Order with Three Factors';
     var progmat1 reason1 verbal1     /* Name the observed vars */
         headlng1 headbrd1 headcir1 bizyg1 height1 weight1
         progmat2 reason2 verbal2
         headlng2 headbrd2 headcir2 bizyg2 height2 weight2;
     lineqs
         /* First the measurement model: Double Measurement */
         progmat1 = Fprogmat + e1,
         reason1  = Freason  + e2,
         verbal1  = Fverbal  + e3,
         headlng1 = Fheadlng + e4,
         headbrd1 = Fheadbrd + e5,
         headcir1 = Fheadcir + e6,
         bizyg1   = Fbizyg   + e7,
         height1  = Fheight  + e8,
         weight1  = Fweight  + e9,
         progmat2 = Fprogmat + e10,
         reason2  = Freason  + e11,
         verbal2  = Fverbal  + e12,
         headlng2 = Fheadlng + e13,
         headbrd2 = Fheadbrd + e14,
         headcir2 = Fheadcir + e15,
         bizyg2   = Fbizyg   + e16,
         height2  = Fheight  + e17,
         weight2  = Fweight  + e18,
         /* Now a factor analysis model for the latent variables. */
         Fprogmat = lambda1 F1 + e19,
         Freason  = lambda2 F1 + e20,
         Fverbal  = lambda3 F1 + e21,
         Fheadlng = lambda4 F2 + e22,
         Fheadbrd = lambda5 F2 + e23,
         Fheadcir = lambda6 F2 + e24,
         Fbizyg   = lambda7 F2 + e25,
         Fheight  = lambda8 F3 + e26,
         Fweight  = lambda9 F3 + e27;
     std     /* Variances (not standard deviations) of exogenous vars */
         F1 F2 F3 = 3 * 1, e1-e27 = 27 * psi:;
     cov F1 F2 = phi12, F1 F3 = phi13, F2 F3 = phi23,
         e1-e9 = 36 * cova:, e10-e18 = 36 * covb:;
     bounds 0 < psi1-psi27, lambda1 > 0, lambda4 > 0, lambda8 > 0 ;
     /* Identified provided phi13 or phi23 is non-zero. */
```

Predicted Model Matrix

		progmat1	reason1	verbal1
progmat1	Progressive matrices 1 (M)	1.0411	0.6069	0.5454
reason1	Reasoning Ability 1 (R)	0.6069	1.0934	0.7595
verbal1	Verbal Ability 1 (V)	0.5454	0.7595	1.0370
headlng1	Head Length 1 (L)	0.0757	0.0716	0.1106
headbrd1	Head breadth 1 (B)	0.0358	0.1912	0.1755

Again the diagonal elements don't equal one.

Chi-Square	102.2085
Chi-Square DF	60
Pr > Chi-Square	0.0006

The fit is better but still the model departs significantly from an acceptable fit.

Rank Order of the 10 Largest Asymptotically Standardized Residuals

Row	Column	Residual
headlng2	headlng1	5.38024
headcir2	headlng1	3.43746
headcir2	headcir1	3.38226
headlng2	headcir1	3.02185
headbrd2	headlng1	-2.43568
headbrd2	headbrd1	2.41706
height2	headlng1	2.36728
headbrd2	verbal1	-2.24997
headbrd1	verbal1	-2.12272
headbrd1	reason1	-2.05315

Big positive residuals mean that the correlation is greater than the model would predict. One cannot discount the possibility of correlated measurement errors between twins. Or maybe an omitted variable?

Four of the top five correlations involve head length. Try dropping it.

```
/************ twinfac6.sas ********************/
%include 'twinread.sas';
TITLE2 'Confirmatory Factor Analysis with Double Measurement';

proc calis corr residual;
     title3 'Second-Order with Three Factors (Drop Head Length)';
     var progmat1 reason1 verbal1     /* Name the observed vars */
         headbrd1 headcir1 bizyg1 height1 weight1
         progmat2 reason2 verbal2
         headbrd2 headcir2 bizyg2 height2 weight2;
     lineqs
         /* First the measurement model: Double Measurement */
         progmat1 = Fprogmat + e1,
         reason1  = Freason  + e2,
         verbal1  = Fverbal  + e3,
         headbrd1 = Fheadbrd + e4,
         headcir1 = Fheadcir + e5,
         bizyg1   = Fbizyg   + e6,
         height1  = Fheight  + e7,
         weight1  = Fweight  + e8,
         progmat2 = Fprogmat + e9,
         reason2  = Freason  + e10,
         verbal2  = Fverbal  + e11,
         headbrd2 = Fheadbrd + e12,
         headcir2 = Fheadcir + e13,
         bizyg2   = Fbizyg   + e14,
         height2  = Fheight  + e15,
         weight2  = Fweight  + e16,
         /* Now a factor analysis model for the latent variables. */
         Fprogmat = lambda1 F1 + e17,
         Freason  = lambda2 F1 + e18,
         Fverbal  = lambda3 F1 + e19,
         Fheadbrd = lambda4 F2 + e20,
         Fheadcir = lambda5 F2 + e21,
         Fbizyg   = lambda6 F2 + e22,
         Fheight  = lambda7 F3 + e23,
         Fweight  = lambda8 F3 + e24;
     std      /* Variances (not standard deviations) of exogenous vars */
         F1 F2 F3 = 3 * 1, e1-e24 = 24 * psi:;
     cov F1 F2 = phi12, F1 F3 = phi13, F2 F3 = phi23,
         e1-e8 = 28 * cova:, e9-e16 = 28 * covb:;
     bounds 0 < psi1-psi24, lambda1 > 0, lambda4 > 0, lambda7 > 0 ;
     /* Identified provided phi13 or phi23 is non-zero. */
```

Finally it fits!

Chi-Square	47.9735
Chi-Square DF	45
Pr > Chi-Square	0.3532

There is still some suggestion of numerical trouble.

Average Absolute Residual	0.049670
Average Off-diagonal Absolute Residual	0.053026

WARNING: Fitting a correlation matrix should lead to insignificant diagonal residuals. The maximum value 0.0842692654 of the diagonal residuals may be too high for a valid Chi-Square test statistic and standard errors.

Rank Order of the 10 Largest Asymptotically Standardized Residuals

Row	Column	Residual
weight2	headcir2	3.07183
height1	headbrd1	-2.26856
height2	headbrd2	-2.25343
headcir2	weight1	2.17202
height2	headbrd1	-1.96011
weight2	verbal1	-1.95759
headbrd2	verbal1	-1.92415
weight2	headbrd2	-1.76589
progmat2	headcir1	1.76384
headbrd1	verbal1	-1.70957

Manifest Variable Equations with Estimates

progmat1 =	0.5861*F1	+	1.0000 e1
Std Err	0.0822 lambda1		
t Value	7.1276		
reason1 =	0.7382*F1	+	1.0000 e2
Std Err	0.0810 lambda2		
t Value	9.1111		
verbal1 =	0.9363*F1	+	1.0000 e3
Std Err	0.0824 lambda3		
t Value	11.3604		
headlng1 =	0.5478*F2	+	1.0000 e4
Std Err	0.0833 lambda4		
t Value	6.5759		
headbrd1 =	0.7547*F2	+	1.0000 e5
Std Err	0.0819 lambda5		
t Value	9.2122		
headcir1 =	0.7566*F2	+	1.0000 e6
Std Err	0.0818 lambda6		
t Value	9.2447		

```
bizyg1   =   0.8634*F2       +  1.0000 e7
Std Err      0.0796 lambda7
t Value     10.8468
height1  =   0.9133*F3       +  1.0000 e8
Std Err      0.0817 lambda8
t Value     11.1850
weight1  =   0.7078*F3       +  1.0000 e9
Std Err      0.0829 lambda9
t Value      8.5416
progmat2 =   0.5861*F1       +  1.0000 e10
Std Err      0.0822 lambda1
t Value      7.1276
reason2  =   0.7382*F1       +  1.0000 e11
Std Err      0.0810 lambda2
t Value      9.1111
verbal2  =   0.9363*F1       +  1.0000 e12
Std Err      0.0824 lambda3
t Value     11.3604
headlng2 =   0.5478*F2       +  1.0000 e13
Std Err      0.0833 lambda4
t Value      6.5759
headbrd2 =   0.7547*F2       +  1.0000 e14
Std Err      0.0819 lambda5
t Value      9.2122
headcir2 =   0.7566*F2       +  1.0000 e15
Std Err      0.0818 lambda6
t Value      9.2447
bizyg2   =   0.8634*F2       +  1.0000 e16
Std Err      0.0796 lambda7
t Value     10.8468
height2  =   0.9133*F3       +  1.0000 e17
Std Err      0.0817 lambda8
t Value     11.1850
weight2  =   0.7078*F3       +  1.0000 e18
Std Err      0.0829 lambda9
t Value      8.5416
```

Variances of Exogenous Variables

Variable	Parameter	Estimate	Standard Error	t Value
F1		1.00000		
F2		1.00000		
F3		1.00000		
e1	psi1	0.69593	0.12434	5.60
e2	psi2	0.49790	0.09841	5.06
e3	psi3	0.14192	0.06224	2.28
e4	psi4	0.66958	0.11785	5.68
e5	psi5	0.41490	0.08494	4.88
e6	psi6	0.44225	0.08890	4.97
e7	psi7	0.22892	0.06424	3.56
e8	psi8	0.14871	0.05415	2.75
e9	psi9	0.50224	0.09498	5.29
e10	psi10	0.64960	0.11413	5.69
e11	psi11	0.45179	0.08817	5.12

e12	psi12	0.18666	0.06402	2.92
e13	psi13	0.71060	0.12338	5.76
e14	psi14	0.43373	0.08525	5.09
e15	psi15	0.41409	0.08260	5.01
e16	psi16	0.26965	0.06617	4.08
e17	psi17	0.14808	0.05314	2.79
e18	psi18	0.48475	0.09056	5.35

Covariances Among Exogenous Variables

Var1	Var2	Parameter	Estimate	Standard Error	t Value
F1	F2	phi12	0.31905	0.11330	2.82
F1	F3	phi13	0.24439	0.11683	2.09
F2	F3	phi23	0.79032	0.05005	15.79
e1	e2	cova1	0.13450	0.08260	1.63
e1	e3	cova2	-0.01906	0.05938	-0.32
e2	e3	cova3	0.05271	0.05946	0.89
e1	e4	cova4	-0.00727	0.08562	-0.08
e2	e4	cova5	-0.09815	0.07606	-1.29
e3	e4	cova6	-0.05613	0.05333	-1.05
e1	e5	cova7	-0.07781	0.07236	-1.08
e2	e5	cova8	-0.05473	0.06480	-0.84
e3	e5	cova9	-0.05777	0.04881	-1.18
e4	e5	cova10	-0.12346	0.07155	-1.73
e1	e6	cova11	-0.00255	0.07422	-0.03
e2	e6	cova12	-0.01052	0.06654	-0.16
e3	e6	cova13	-0.03013	0.04981	-0.60
e4	e6	cova14	0.41046	0.09084	4.52
e5	e6	cova15	0.07924	0.06647	1.19
e1	e7	cova16	-0.16285	0.06199	-2.63
e2	e7	cova17	-0.05027	0.05532	-0.91
e3	e7	cova18	-0.07697	0.04511	-1.71
e4	e7	cova19	-0.04005	0.05989	-0.67
e5	e7	cova20	0.07576	0.05872	1.29
e6	e7	cova21	0.02192	0.05767	0.38
e1	e8	cova22	-0.08361	0.05305	-1.58
e2	e8	cova23	-0.03826	0.04896	-0.78
e3	e8	cova24	-0.03894	0.04149	-0.94
e4	e8	cova25	0.12127	0.05451	2.22
e5	e8	cova26	-0.06573	0.04881	-1.35
e6	e8	cova27	0.04241	0.05018	0.85
e7	e8	cova28	0.0003496	0.04591	0.01
e1	e9	cova29	-0.05932	0.07665	-0.77
e2	e9	cova30	0.06451	0.06884	0.94
e3	e9	cova31	0.00692	0.04993	0.14
e4	e9	cova32	0.22629	0.08014	2.82
e5	e9	cova33	0.07889	0.06564	1.20
e6	e9	cova34	0.25423	0.07366	3.45
e7	e9	cova35	0.17200	0.06010	2.86
e8	e9	cova36	-0.01219	0.05211	-0.23
e10	e11	covb1	0.04963	0.07290	0.68
e10	e12	covb2	0.05511	0.05882	0.94
e11	e12	covb3	0.06858	0.05866	1.17

etc.