STA 413F2011 Assignment 11

1. Let $X_1, \ldots, X_{n_1}$ be a random sample from a distribution with density $f(x|\theta)$, and we seek to test $H_0 : \theta = \theta_0$ for large n. Writing $G_n = -2 \ln \Lambda$, show $G_n \stackrel{d}{\rightarrow} G \sim \chi^2(1)$. The critical step is to expand $\ell(\theta_0)$ about $\widehat{\theta}_n$. You may assume that $-\frac{1}{n}\ell''(\theta_n^*) \stackrel{p}{\rightarrow} I(\theta_0)$, which is true and believable, but the details are a bit too tricky.

2. Let $X_1, \ldots, X_{n_1}$ be a random sample from a Bernoulli distribution with parameter θ_1, and independently of $X_1, \ldots, X_{n_1}$, let $Y_1, \ldots, Y_{n_2}$ be a random sample from a Bernoulli distribution with parameter θ_2. Write down and simplify the test statistic for large-sample likelihood ratio test of $H_0 : \theta_1 = \theta_2$. Show your work, except you can skip derivation of the MLEs if you're sure you know how to do it.

3. Of a random sample of 150 Special Needs students in the Toronto District School Board, 19 were in regular classes, and the rest were in Special Education classes. Of a random sample of 200 Special Needs students in the Toronto Separate School Board, 48 were in regular classes, and the rest were in Special Education classes. Test for difference between the proportions of Special Needs students in regular classes in the two school boards. Use $\alpha = 0.01$. What do you conclude? Is the proportion of Special Needs students greater in one of the school boards? Of course you should use the test from the last question.

4. Let $X_1, \ldots, X_{n_1}$ be a random sample from a distribution with density
$$f(x|\mu, \tau) = \sqrt{\frac{\tau}{2\pi}} e^{-\frac{\tau}{2}(x-\mu)^2}.$$
You should be able to just write down the MLE $\widehat{\tau}$ without doing any work, and say how you know.

5. Let $X_1, \ldots, X_{n_1}$ be a random sample from a distribution with density
$$f(x|\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}} I(x > 0),$$
where $\theta > 0$. A sample of size 50 yields $\overline{X} = 3.5$. Carry out a large-sample likelihood ratio test of $H_0 : \theta = 5$. Do you reject H_0 at $\alpha = 0.05$? Answer Yes or No.

6. Let $X_1, \ldots, X_{n_1}$ be a random sample from a distribution with density
$$f(x|\theta) = \theta e^{-\theta x} I(x > 0),$$
where $\theta > 0$. A sample of size 50 yields $\overline{X} = 3.5$. Carry out a large-sample likelihood ratio test of $H_0 : \theta = 0.20$. Do you reject H_0 at $\alpha = 0.05$? Answer Yes or No.

7. Let $X_1, \ldots, X_{n_1}$ be a random sample from a Poisson distribution with parameter λ. Using the *definition* of sufficiency, show that $Y = \sum_{i=1}^n X_i$ is sufficient for λ.

8. Let $X_1, \ldots, X_{n_1}$ be a random sample from a Bernoulli distribution with parameter θ. Using the *definition* of sufficiency, show that $Y = \sum_{i=1}^n X_i$ is sufficient for θ.

9. Read Section 7.2. Using the factorization Theorem, do Exercises 7.2.1, 7.2.2, 7.2.4, 7.2.5, 7.2.6 and 7.2.7.

Bring a calculator to the quiz!