STA 413F2008 Assignment 3: A Little Basic Math

This assignment is based on material that you probably know already. However, the notation used in Statistics can be an obstacle for some students, so we will review the following basic rules.

- The distributive law: $a(b+c) = ab + ac$. You may see this in a form like

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} \theta x_i$$

- Power of a product is the product of powers: $(ab)^c = a^c\, b^c$. You may see this in a form like

$$\left(\prod_{i=1}^{n} x_i \right)^{\alpha} = \prod_{i=1}^{n} x_i^{\alpha}$$

- Multiplication is addition of exponents: $a^b a^c = a^{b+c}$. You may see this in a form like

$$\prod_{i=1}^{n} \theta e^{-\theta x_i} = \theta^n \exp(-\theta \sum_{i=1}^{n} x_i)$$

- Powering is multiplication of exponents: $(a^b)^c = a^{bc}$. You may see this in a form like

$$(e^{\mu t + \frac{1}{2}\sigma^2 t^2})^n = e^{n\mu t + \frac{1}{2} n \sigma^2 t^2}$$

- Log of a product is sum of logs: $\ln(ab) = \ln(a) + \ln(b)$. You may see this in a form like

$$\ln \prod_{i=1}^{n} x_i = \sum_{i=1}^{n} \ln x_i$$

- Log of a power is the exponent times the log: $\ln(a^b) = b\,\ln(a)$. You may see this in a form like

$$\ln(\theta^n) = n \ln \theta$$

- The log is the inverse of the exponential function: $\ln(e^a) = a$. You may see this in a form like

$$\ln \left(\theta^n \exp(-\theta \sum_{i=1}^{n} x_i) \right) = n \ln \theta - \theta \sum_{i=1}^{n} x_i$$

Choose the correct answer.

1. $\prod_{i=1}^n e^{x_i} =$
   (a) $\exp(\prod_{i=1}^n x_i)$
   (b) e^{nx_i}
   (c) $\exp(\sum_{i=1}^n x_i)$

2. $\prod_{i=1}^n \lambda e^{-\lambda x_i} =$
   (a) $\lambda e^{-\lambda^n x_i}$
   (b) $\lambda^n e^{-\lambda n x_i}$
   (c) $\lambda^n \exp(-\lambda \sum_{i=1}^n x_i)$
   (d) $\lambda^n \exp(-n\lambda \sum_{i=1}^n x_i)$
   (e) $\lambda^n \exp(-\lambda^n \sum_{i=1}^n x_i)$

3. $\prod_{i=1}^n a_i^b =$
   (a) na^b
   (b) a^{nb}
   (c) $(\prod_{i=1}^n a_i)^b$

4. $\prod_{i=1}^n a^{b_i} =$
   (a) na^{b_i}
   (b) a^{nb_i}
   (c) $\sum_{i=1}^n a^{b_i}$
   (d) $a^{\prod_{i=1}^n b_i}$
   (e) $a^{\sum_{i=1}^n b_i}$

5. $\left(e^{\lambda(e^t-1)}\right)^n =$
   (a) $ne^{\lambda(e^t-1)}$
   (b) $e^{n\lambda(e^t-1)}$
   (c) $e^{\lambda(e^{nt}-1)}$
   (d) $e^{n\lambda(e^t-n)}$

6. $\left(\prod_{i=1}^n e^{-\lambda x_i}\right)^2 =$
   (a) $e^{-2n\lambda x_i}$
   (b) $e^{-2\lambda \sum_{i=1}^n x_i}$
   (c) $2e^{-\lambda \sum_{i=1}^n x_i}$

7. True, or False?

   (a) $\sum_{i=1}^{n} \frac{1}{x_i} = \frac{1}{\sum_{i=1}^{n} x_i}$
   (b) $\prod_{i=1}^{n} \frac{1}{x_i} = \frac{1}{\prod_{i=1}^{n} x_i}$
   (c) $\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$
   (d) $\ln(a+b) = \ln(a) + \ln(b)$
   (e) $e^{a+b} = e^a + e^b$
   (f) $e^{a+b} = e^a e^b$
   (g) $e^{ab} = e^a e^b$
   (h) $\prod_{i=1}^{n}(x_i + y_i) = \prod_{i=1}^{n} x_i + \prod_{i=1}^{n} y_i$
   (i) $\ln(\prod_{i=1}^{n} a_i^b) = b \sum_{i=1}^{n} \ln(a_i)$
   (j) $\sum_{i=1}^{n} \prod_{j=1}^{n} a_j = n \prod_{j=1}^{n} a_j$
   (k) $\sum_{i=1}^{n} \prod_{j=1}^{n} a_i = \sum_{i=1}^{n} a_i^n$
   (l) $\sum_{i=1}^{n} \prod_{j=1}^{n} a_{i,j} = \prod_{j=1}^{n} \sum_{i=1}^{n} a_{i,j}$

8. Simplify as much as possible.

   (a) $\ln \prod_{i=1}^{n} \theta^{x_i}(1-\theta)^{1-x_i}$
   (b) $\ln \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x}(1-\theta)^{m-x_i}$
   (c) $\ln \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$
   (d) $\ln \prod_{i=1}^{n} \theta(1-\theta)^{x_i-1}$
   (e) $\ln \prod_{i=1}^{n} \frac{1}{\theta} e^{-x_i/\theta}$
   (f) $\ln \prod_{i=1}^{n} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} e^{-x_i/\beta} x_i^{\alpha-1}$
   (g) $\ln \prod_{i=1}^{n} \frac{1}{2^{\nu/2}\Gamma(\nu/2)} e^{-x_i/2} x_i^{\nu/2-1}$
   (h) $\ln \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$
   (i) $\prod_{i=1}^{n} \frac{1}{\beta-\alpha} I(\alpha \le x_i \le \beta)$ (Express in terms of the minimum and maximum y_1 and y_n.)