STA 347F2003 Quiz 9

1. (30 Points) Typographical errors in a document occur according to a Poisson process with rate $\lambda = 2$ per page. Given that the first page contained three errors, what is the probability of exactly ten errors in the first six pages?

2. (30 Points) Given $L = \lambda$, the random variable X has a Poisson distribution with parameter λ. The continuous random variable L has density $f_L(\lambda) = e^{-\lambda}\mathbf{1}\{x \geq 0\}$. Find the marginal probability $Pr\{X = x\}$. Circle your answer.

3. Visits to a Web site occur according to a Poisson process with rate λ per hour.

   (a) (30 Points) Given n visits in the first hour, find the probability that k of them occurred in the interval $(0, p]$, where $0 < p < 1$ (so p is a fraction of an hour).

   (b) (10 Points) The conditional distribution from question (3a) has a name. What is it?

STA 347F2003 Formula Sheet 2

If $0 < a < 1$ then $\sum_{k=j}^{\infty} a^k = \frac{a^j}{1-a}$.

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{k=0}^{\infty} \frac{x^k}{k!}$.

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k \, b^{n-k}$.

Binomial: $X \sim B(n, p)$ means $Pr\{X = k\} = \binom{n}{k} p^k \, (1-p)^{n-k}$ for $k = 0, , \ldots , n$. Note: $E[X] = np$, $V[X] = np(1-p)$.

Poisson: $X \sim P(\mu)$ means $Pr\{X = k\} = \frac{e^{-\mu}\mu^k}{k!}$ for $k = 0, 1, \ldots$. Note: $E[X] = V[X] = \mu$

Exponential: $X \sim \exp(\lambda)$ means $f_X(x) = \lambda \, e^{-\lambda x}\mathbf{1}\{x \geq 0\}$. Note $E[X] = 1/\lambda$, and $F_X(x) = (1 - e^{-\lambda x})\mathbf{1}\{x \geq 0\}$.

Gamma: $X \sim G(n, \lambda)$ means $f_X(x) = \frac{\lambda^n}{(n-1)!} \, e^{-\lambda x} x^{n-1}\mathbf{1}\{x \geq 0\}$. Note $E[X] = \frac{\lambda}{n}$. The sum of n independent exponential random variables is Gamma.

Jerry's Answers to Quiz 9

(1) $\Pr\{X(6)=10 \mid X(1)=3\} = \dfrac{\Pr\{N(0,1]=3, N(0,6]=10\}}{\Pr\{N(0,1]=3\}}$

$$= \frac{\Pr\{N(0,1]=3, N(1,6]=7\}}{\Pr\{N(0,1]=3\}} = \frac{\cancel{\Pr\{N(0,1]=3\}}\Pr\{N(1,6]=7\}}{\cancel{\Pr\{N(0,1]=3\}}}$$

$$= \frac{e^{-(2)(5)}\,[(2)(5)]^7}{7!} = \frac{e^{-10}\,10^7}{7!} \approx .09$$

(2) $\Pr\{X=x\} = \displaystyle\int_0^\infty \Pr\{X=x \mid L=\lambda\} f_L(\lambda)\,d\lambda$

$$= \int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!} e^{-\lambda}\,d\lambda = \int_0^\infty \frac{1}{x!} e^{-2\lambda} \lambda^{(x+1)-1}\,d\lambda$$

$$= \frac{1}{2^{x+1}} \underbrace{\int_0^\infty \frac{2^{x+1}}{([x+1]-1)!} e^{-2\lambda} \lambda^{(x+1)-1}\,d\lambda}_{=1}$$

$$= \frac{1}{2^{x+1}} \quad \text{for } x = 0, 1, \ldots$$

(3) $$\Pr\{N(0,p]=k \mid N(0,1]=n\} = \frac{\Pr\{N(0,p]=k,\ N(0,1]=n\}}{\Pr\{N(0,1]=n\}}$$

$$= \frac{\Pr\{N(0,p]=k,\ N(p,1]=n-k\}}{\Pr\{N(0,1]=n\}}$$

$$= \frac{\Pr\{N(0,p]=k\}\Pr\{N(p,1]=n-k\}}{\Pr\{N(0,1]=n\}}$$

$$= \frac{\dfrac{e^{-\lambda p}(\lambda p)^k}{k!}\ \dfrac{e^{-\lambda(1-p)}[\lambda(1-p)]^{n-k}}{(n-k)!}}{\dfrac{e^{-\lambda}\lambda^n}{n!}}$$

$$= \frac{n!}{k!\,(n-k)!}\ \frac{\cancel{\lambda^{k+n-k}}\ p^k(1-p)^{n-k}}{\cancel{\lambda^n}}$$

$$= \binom{n}{k} p^k (1-p)^{n-k}.$$ BINOMIAL