STA 347F2003 Quiz 3

1. Let $X_0, X_1, \ldots$ be a stationary Markov chain with transition matrix

$$\mathbf{P} = \begin{array}{c||c|c|c|} & 0 & 1 & 2 \\ \hline\hline 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 1 & 0 \\ \hline 2 & a & b & c \\ \hline \end{array}$$

   (a) (20 Points) What is $\mathbf{P}^2$?

   (b) (5 Points) What is $Pr\{X_2 = 2 | X_0 = 2\}$?

   (c) (10 Points) What is $Pr\{X_3 = 1 | X_0 = 2\}$? Show some work.

   (d) (10 Points) Suppose $\mathbf{p}^{(0)} = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$. What is $Pr\{X_2 = 1\}$? Show some work.

   (e) Suppose $\mathbf{p}^{(0)} = [\frac{1}{2}, \frac{1}{2}, 0]$.

      i. (10 Points) What is $Pr\{X_2 = 0\}$? Show some work.

      ii. (5 Points) What is $Pr\{X_2 = 1\}$? Show some work.

      iii. (15 Points) What is $Pr\{X_{25} = 1\}$? Just write down the answer.

2. (25 Points) Let $X_0, X_1, \ldots$ be a stationary Markov chain with transition matrix

$$\mathbf{P} = \begin{array}{c||c|c|} & 0 & 1 \\ \hline\hline 0 & a & 1-a \\ \hline 1 & b & 1-b \\ \hline \end{array}$$

   Then $Z_n = (X_{n-1}, X_n)$ is a Markov chain having the four states (0,0), (0,1), (1,0), (1,1). Give its transition matrix.

Jerry's Answers to Quiz 3

(1) a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & c \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a+ac & b+bc & c^2 \end{bmatrix}$$

b) $Pr\{X_2 = 2 \mid X_0 = 2\} = c^2$

c) $Pr\{X_3 = 1 \mid X_0 = 2\} = (a)(0) + (b)(1) + cb(c+1)$
$= b(1 + c + c^2)$

d) $Pr\{X_2 = 1\} = \left(\frac{1}{3}\right)(0) + \left(\frac{1}{3}\right)(1) + \left(\frac{1}{3}\right)(b+bc) = \frac{1}{3}(1+b+bc)$

e) (i) $Pr\{X_2 = 0\} = \left(\frac{1}{2}\right)(1) + \left(\frac{1}{2}\right)(0) + (0)(a+ac) = \frac{1}{2}$

(ii) $Pr\{X_2 = 1\} = \left(\frac{1}{2}\right)(0) + \left(\frac{1}{2}\right)(1) + (0)(b+bc) = \frac{1}{2}$

(iii) $Pr\{X_{25} = 1\} = \frac{1}{2}$

(2)

	(0,0)	(0,1)	(1,0)	(1,1)
(0,0)	a	1 − a	0	0
(0,1)	0	0	b	1 − b
(1,0)	a	1 − a	0	0
(1,1)	0	0	b	1 − b