STA 347F2003 Assignment 11

These are some extra questions for the final exam, covering the time between Assignment 10 and the end of lecture. It is not to be handed in.

1. Read Section 4 in Chapter 5, especially pages 297-301. Do not worry about the proof of Theorem 4.1, but the discussion before it is important, except the stuff about Δw. Sections 4.1 (Shot Noise) and 4.2 (Sum quota sampling) are *not* going to be on the final exam. Do Exercises 4.1, 4.2, 4.3, and 4.5. Also do Problem 4.8.

2. Let N be a discrete random variable taking values $\{1, \ldots\}$, and let $Y_1, Y_2, \ldots$ be independent and identically distributed random variables that are also independent of N, with $E[Y_k] = \mu$ and $Var(Y_k) = \sigma^2$. Let $Z_N = \sum_{k=1}^{N} Y_k$; note that the problem has been changed to make N start at one instead of zero.

   (a) Find $E[Z_N]$.

   (b) Find $Var[Z_N]$.

   (c) Find $F_{Z_N}(z)$.

   (d) Assume $Y_1, Y_2, \ldots$ are continuous. Find the density $f_{Z_N}(z)$. You may move the derivative sign through the infinite sum without comment (it's okay by a dominated convergence argument).

   (e) Let $Y_1, Y_2, \ldots$ be independent exponential random variables with parameter λ, and let N have a geometric distribution (see updated formula sheet).

      i. What is $E[Z_N]$?
      ii. Find the probability density function of Z_N.

3. Do exercise 6.2 on Page 328.