STA 347F2000 Quiz 5

Print your name and student number *neatly* on the first sheet.

1. (30 Points) Let $X_0, X_1, \ldots$ be a regular stationary Markov chain with state space $\{0, \ldots, N\}$, so that $\lim_{n\to\infty} P_{i,j}^{(n)} = \pi_j$ for $j = 0, \ldots, N$, and $\sum_{j=0}^{N} \pi_j = 1$. Use the existence of these limiting probabilities to show that the $1 \times (N+1)$ matrix $\boldsymbol{\pi} = (\pi_0, \ldots, \pi_N)$ satisfies $\boldsymbol{\pi} = \boldsymbol{\pi}\boldsymbol{P}$.

2. On a tropical resort island, the chance of rain tomorrow depend on previous weather conditions only through whether it is raining today. Suppose that if it rains today, it will rain again tomorrow with probability 0.5; if it does not rain today, then it will rain tomorrow with probability 0.1. Let $X_n = \mathbf{1}\{\text{Rain on day } n\}$.

   (a) (10 Points) What is the transition probability matrix of this Markov chain?

   (b) (20 Points) What is the long-run probability of rain?

   (c) (15 Points) Suppose a water park makes an average of \$1,800 a day if it does not rain, and loses \$3,600 if it does rain. What is the long-run daily expected profit? Note: a negative answer would indicate an expected loss.

3. (25 Points) Let $X_0, X_1, \ldots$ be a stationary Markov chain with this transition matrix.

	0	1	2	3
0	0	p	0	$1-p$
1	$1-p$	0	p	0
2	0	$1-p$	0	p
3	p	0	$1-p$	0

   Is this Markov chain regular? Answer **Yes** or **No**. Circle your answer. Justify your answer. No marks without a correct justification.