STA 347F2000 Quiz 2

Print your name and student number *neatly* on the first sheet.

1. (25 points) Let $p_{XY} = \frac{xy}{25}\mathbf{1}\{x = 1,2,3,4\}\mathbf{1}\{y = x,\ldots,3\}$. Find $E[Y|X=2]$. Show some work, but not more than you need to find the answer. This is the easiest question; it is not very long, for the number of points.

2. (25 points) Suppose that the distribution function F_X is differentiable at x. Show $Pr\{x < X \leq x + \triangle x\} = f(x)\,\triangle x + o(\triangle x)$ as $\triangle x \downarrow 0$ implies $F'(x) = f(x)$.

3. A jar contains two objects, a fair die and a biased coin with probability of a Head equal to $\frac{1}{3}$. First you pick an object at random (equally likely). If it is the die, you roll it and count the number of dots. If it is a coin, you toss it repeatedly until the first head appears, and count the number of tails that occur *before* the first head. Let X denote the number resulting from this two-stage experiment.

   (a) (10 Points) Find $Pr(X = 1)$.

   (b) (15 Points) Find $E[X]$. Show your work and **circle your final answer**. Note: You may use the formula $\frac{1-p}{p}$ for the expected value of a geometric random variable; no proof is required. However, **the answer to this question is a number**. If the symbol p occurs anywhere in the final answer you circle, you can get no more than 8 out of the 15 points.

4. (25 points) Let X_n have a binomial distribution with parameters n and p. Let $n \to \infty$ and $p \to 0$ in such a way that the product $np = \lambda$ remains constant. Find the limiting probability mass function of X_n. You may use $\lim_{n\to\infty}(1 + \frac{y}{n})^n = e^y$ and other standard results about limits without proof. Note: the support of the limiting random variable is worth 10 points out of 25.