Name ____________________________

Student Number __________________

Test 3
STA 347s 1991
Erindale College

1. (20 pts) A rich person has a home in the city, a home in the country and three automobiles. When she travels from one home to the other, she either drives or takes a taxi cab. If an automobile is available at the house where she is, she drives with probability p $(0<p<1)$, or takes a taxi with probability $1-p$, independently of past choices and independently of the number of automobiles at her current location. If all her automobiles are at the other location, of course she takes a taxi. Let X_n represent the number of automobiles at her current location (where she is now). Give the matrix of transition probabilities for this Markov Chain.

2. (20 pts) A bank has three tellers. When customer D walks into the bank, he finds that three other customers (call them A, B and C) are already being served. If service times are independent exponential random variables with the same parameter λ, what is the probability that A will finish before D?

a) What is the answer? (This is worth 10 points)

b) Why? (This is worth 10 points)

3. (20 points) Let X_1 and X_2 be independent exponential random variables with $E(X_1)=1/\lambda_1$ and $E(X_2)=1/\lambda_2$, and let $Y=\text{Minimum}(X_1,X_2)$. Derive the conditional distribution of Y given that $Y=X_1$; that is find $P(Y \leq y \mid Y=X_1)$. You may use the fact that $P(X_1<X_2)=\lambda_1/(\lambda_1+\lambda_2)$.

4. (20 points) Definition (3.2) of a Poisson process states that $\{N(t):t\geq 0\}$ satisfies (i) $N(0)=0$, (ii) Stationary and independent increments, (iii) $P\{N(h)=1\} = \lambda h + o(h)$, and (iv) $P\{N(h)\geq 2\} = o(h)$. Denoting by $P_0(t)$ the probability of zero events in $(0,t]$, derive a differential equation for $P_0(t)$. <u>You do NOT need to solve the differential equation</u>.

5. Assume that in Detroit Michigan, murders happen at a Poisson rate of 2 per day.

a) (5 points) What is the probability of waiting more than 12 hours between murders? (Remember the time units are 24 hour days)

b) (5 points) What is the probability of exactly three murders in four days?

c) (5 points) What is the expected number of murders on Saturday night between midnight and 6 am? (Remember the time units are days)

d) (5 points) Explain why a Poisson process might <u>not</u> be a reasonable model for murders in a large city.