

Sample Questions: Survival and Hazard Functions

STA312 Spring 2019. Copyright information is at the end of the last page.

For all these questions, T is a continuous random variable with $P(T > 0) = 1$, density $f(t)$ and cumulative distribution function $F(t) = P(T \leq t)$.

1. The survival function is $S(t) = P(T > t)$. Prove $E(T) = \int_0^{\infty} S(t) dt$.

2. The hazard function is defined by $h(t) = \lim_{\Delta \rightarrow 0} \frac{P(t < T < t + \Delta | T > t)}{\Delta}$,
where $\Delta > 0$. Prove $h(t) = \frac{f(t)}{S(t)}$.

3. Prove $S(t) = e^{-\int_0^t h(x) dx}$.

4. Let $T \sim \exp(\lambda)$. Find the hazard function $h(t)$ for $t > 0$.

5. Let T have the Pareto density $f(t|\theta) = \begin{cases} \frac{\theta}{t^{\theta+1}} & \text{for } t \geq 1 \\ 0 & \text{for } t < 1 \end{cases}$

(a) Find the hazard function $h(t)$ for $t > 1$.

(b) Earlier, we found the MLE $\hat{\theta}_n = \frac{n}{\sum_{i=1}^n \log t_i}$, and $\hat{v}_n = \frac{\hat{\theta}_n^2}{n}$.

i. Give $\widehat{h}(t)$, the maximum likelihood estimate of the hazard function evaluated at a particular time $t > 1$. Your answer is a formula involving t and $\hat{\theta}_n$.

ii. We want a confidence interval for $h(t)$, the hazard function evaluated at a particular time $t > 1$. Give formulas for the lower and upper 95% confidence limits. Show your work.

6. Let T have a gamma distribution with parameters $\alpha > 0$ and $\lambda > 0$.

(a) What is the hazard function?

(b) Using R, plot the hazard function for several values of α and λ . How do the parameter values influence the shape of the hazard function?

This assignment was prepared by [Jerry Brunner](#), Department of Mathematical and Computational Sciences, University of Toronto. It is licensed under a [Creative Commons Attribution - ShareAlike 3.0 Unported License](#). Use any part of it as you like and share the result freely. The L^AT_EX source code is available from the course website:

<http://www.utstat.toronto.edu/~brunner/oldclass/312s19>