# Survival and Hazard Functions[1]

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[1] See last slide for copyright information.

# Background Reading

Chapter Two in *Applied Survival Analysis Using R*

# Survival Function

Let $T$ be a continuous random variable with $P(T > 0) = 1$. It will represent time to failure. The *survival function* is the probability of lasting past time $t$.

$$\begin{aligned} S(t) &\stackrel{def}{=} P(T > t) \\ &= 1 - F(t) \\ &= \int_t^\infty f(x)\, dx \end{aligned}$$

# The hazard function $h(t)$

- Idea: The probability of dying at time $t$ given that you have lived this long.
- But the probability of dying at exactly time $t$ is zero.
- So consider the probability of dying in in the next *instant* following $t$, given that you have lived to time $t$.
- The meaning of *instant* is a tiny interval of time, length $\Delta$.

$$P(t < T < t + \Delta | T > t)$$

- This clearly depends on the length of the time interval $\Delta$. So make the conditional probability *relative* to the length of time $\Delta$.

$$\frac{P(t < T < t + \Delta | T > t)}{\Delta}$$

- To capture the idea of a *really* tiny interval, let $\Delta \rightarrow 0$. This leads to the definition

$$h(t) \stackrel{def}{=} \lim_{\Delta \rightarrow 0} \frac{P(t < T < t + \Delta | T > t)}{\Delta}$$

# The Hazard Function

$$h(t) \overset{def}{=} \lim_{\Delta \to 0} \frac{P(t < T < t + \Delta | T > t)}{\Delta}$$

where $\Delta > 0$.

It will be shown that

$$h(t) = \frac{f(t)}{S(t)}$$

# The Cumulative Hazard Function

$$
\begin{aligned}
H(t) &= \int_0^t h(x)\,dx \\
&\overset{show}{=} -\log S(t)
\end{aligned}
$$

So that

$$S(t) = e^{-\int_0^t h(x)\,dx}$$

# Density, Cumulative Distribution Function, Survival Function and Hazard Function are all Equivalent

$$
\begin{aligned}
f(t) &= F'(t) \\
F(t) &= \int_{-\infty}^{t} f(x)\, dx \\
S(t) &= 1 - F(t) \\
h(t) &= \frac{f(t)}{S(t)} \\
S(t) &= e^{-\int_0^t h(x)\, dx}
\end{aligned}
$$

- With one function, it is easy to obtain the others.
- We will focus on the hazard function.

# Copyright Information